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Exact results for supersymmetric renormalization and the supersymmetric flavor problem

Ann E. Nelson
Department of Physics, Box 1560, University of Washington
Seattle, WA 98195-1560, USA
E-mail: anelson@fermi.phys.washington.edu

Matthew J. Strassler
Department of Physics and Astronomy, University of Pennsylvania
209 S. 33rd St., Philadephia, PA 19104, USA
E-mail: strassler@physics.upenn.edu

Abstract: We explore the effects of a strongly-coupled, approximately scale-invariant sector on the renormalization of soft supersymmetry breaking terms. A useful formalism for deriving exact results for renormalization of soft supersymmetry breaking terms is given in an appendix, and used to generalize previously known results to include the effects of nontrilinear superpotential terms. We show that a class of theories which explain the flavor hierarchy without flavor symmetries can also solve the supersymmetric flavor problem by producing nearly degenerate masses for the first two generations of scalar superpartners within each charge sector. Effects from trilinear scalar terms are also suppressed, although their initial values must be relatively small. Our mechanism results in testable predictions for the superpartner spectrum.

Keywords: Supersymmetry Breaking, Beyond Standard Model.

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Contents

1. Introduction

Recently, we have shown that nearly-conformal strongly coupled supersymmetric theories can explain the striking features of the quark and lepton masses and mixing parameters, without flavor symmetry [1]. We also pointed out that this mechanism for flavor could in some cases simultaneously solve the supersymmetric flavor problem. In this note we consider in more detail the effects of such theories on soft supersymmetry breaking terms. Exact results for the renormalization of such terms show that under certain conditions, these theories may also suppress flavor changing neutral currents. Our results for the running of soft SUSY-breaking terms near supersymmetric fixed points have testable implications for the supersymmetric flavor problem and the spectrum of superpartners. In a lengthy appendix, we introduce a unifying formalism for the derivation of the necessary results, and generalize previous results [2]–[14] to include the effects of nontrilinear superpotential couplings.

2. Summary of scenario for flavor hierarchy, and effects on soft terms

We begin with a review of the class of models introduced in ref. [1] to explain the hierarchy of fermion masses and mixing parameters.

Besides the standard model gauge group, S, (possibly embedded in a grand unified gauge group), we assume another (not necessarily simple) gauge group G, which we call the "conformal sector". The matter content includes standard model fields X charged under S, and additional fields Q, charged under G. Some Q fields must also carry standard model charges.

It is the strong dynamics of G which will generate the flavor hierarchy. In ref. [1], we argued that in order to produce a fermion mass hierarchy of order 10^5, G must be strongly coupled over a large energy range. This is possible if the strong couplings of the theory are in the vicinity of an approximate infrared conformal fixed point (CFP).

In order to escape the strong dynamics of G at low energies, the theory must also contain some relevant term, which is small at short distance, but which eventually drives the theory away from the CFP. This will sometimes require an additional gauge group H with additional matter; we will call this the "escape sector".

Our scenario contains several energy regimes. There may or may not be a "pre-conformal regime" between the Planck/string scale and a scale $M_>$, in which the conformal sector has not yet reached the vicinity of a CFP. In this regime the dynamics may be very complicated and we know little about it. Between the scales $M_>$ and $M_<$ the theory is in a "conformal regime"; all strong couplings are nearly scale invariant, any relevant couplings are very small, and the couplings of the standard model are perturbative. There is then an "escape" from the conformal regime at the scale $M_<$. Below $M_<$ we are in the "SSM" regime; the low energy theory is simply a supersymmetric standard model with the special properties we will outline below. This regime survives down to the TeV scale.

Renormalization generates the flavor hierarchy. There are no flavor symmetries, or flavor hierarchies, at the Planck or string scale. Our key assumptions are (a) that all standard model Yukawa couplings at the Planck scale are allowed and are of order 1, with no special relations among them, and (b) that all soft SUSY-breaking masses in the standard model sector are generated above or near the top of the conformal regime, and are roughly of the same order, again with no special relations among them.

The essential results of this paper stem from the features of renormalization in both the conformal and SSM regimes. In particular, the conformal regime suppresses most (though in general not all) soft SUSY-breaking parameters, while the SSM enhances some of them, sometimes in flavor-blind fashion. These two facts will make it possible to wash out much of the flavor dependence inherent in the SUSY-breaking parameters of hidden-sector models.

More specifically, the conformal regime does the following:

1. It sets a number of supersymmetric couplings to fixed, strong values, and drives their supersymmetry-breaking higher components toward zero.

2. It generates large anomalous dimensions, automatically positive, for certain standard model matter fields to which it is coupled in the superpotential.

3. Some superpotential couplings, including certain Yukawa couplings of standard model fermions to the Higgs boson, run toward zero as a power of the renormalization scale, along with the corresponding A-terms. In particular, the fermions of the first two generations and part of the third have their Yukawa couplings and trilinear scalar couplings driven small.

4. Various linear combinations of scalar masses-squared also run towards zero as a power of the renormalization scale. In some models, this guarantees small masses for the first two generations of scalar superpartners at the scale $M_<$. These masses will not quite reach zero due to small flavor-dependent violations of conformal symmetry from the standard model interactions, and so the first two generations of sfermions have small non-degenerate masses at $M_<$.

5. Other standard model couplings and soft SUSY-breaking parameters are relatively unaffected. In particular, the gaugino masses, and masses and A-terms of some or all of the third generation scalars, remain substantial.

After the escape from the conformal regime, the theory enters the SSM regime, in which

1. The standard model Yukawa couplings and most of the soft parameters undergo modest multiplicative renormalizations; the structure of the Yukawa couplings and A-terms is not changed significantly.

2. The soft masses for light-generation sleptons and squarks, driven small in the conformal regime, receive large, positive, and flavor-blind additive renormalizations to their masses-squared. By the TeV scale, these dominate the flavor-violating masses-squared which were present at $M_<$, leaving these sleptons and squarks with masses that are nearly degenerate in each charge sector.

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3. The soft scalar masses of at least part of the third generation, including at least the top and left-handed bottom squarks, and the Higgs doublets, remain of order $M_{\rm SUSY}$, as do the A-terms for third generation scalars. We have no predictions for these quantities, and take them to be free parameters.

These two regimes combine together to suppress flavor changing neutral currents. The spectrum of the model is distinctive. The sleptons and squarks of the first two generations, and perhaps part of the third, show degeneracy in each charge sector, explaining the absence of large flavor changing neutral currents from virtual superpartner exchange. As we will see, there are definite predictions for certain combinations of these masses as a function of the gaugino masses and the scale $M_<$.

The running in the conformal regime ensures that the A-terms of the light generations, potential additional sources of flavor violation, will also end up small — roughly proportional to the corresponding Yukawa couplings. However, such A-terms are problematic [15] for $\mu \to e\gamma$ and for electric dipole moments. If our scenario is correct, then there is some additional overall suppression of trilinear couplings. It is likely we will see observable flavor and CP-violating effects in the relatively near future.

3. The conformal regime

Many of our claims about the conformal regime have already been given in [1]. We argued that among all possible superpotential terms coupling standard model fields to the conformal sector, only terms *linear* in the standard model fields could be nonvanishing at the fixed point, with all others being irrelevant. (This is the reason that some conformal sector fields Q must also be charged under the standard model for our scenario to operate.) Those standard model fields X which couple in this way to the conformal sector develop positive anomalous dimensions of order one. These anomalous scalings suppress all of the other couplings of the X fields, such as their Yukawa couplings to the Higgs bosons.

In a superconformal field theory, there is a U(1) R-symmetry which determines operator dimensions, including the dimensions of the standard model fields. In our scenario, the conformal symmetry only occurs below the gravitational scales, so the R-symmetry is an accidental symmetry determined by the near-conformal dynamics. Within each standard-model charge sector — say, among the three electroweak-singlet leptons — distinct R-charges r_i will be held by three linearly independent combinations of the three fields with that standard model charge. These three combinations are determined dynamically and cannot be simply read off from the lagrangian at the Planck scale. Each combination has a distinct dimension $d_i = 3r_i/2$ in the conformal regime and cannot mix with the others; consequently, its Yukawa couplings will all be reduced by a distinct suppression factor, namely $(M_</M_>)^{d_i-1}$. Thus the algebraic hierarchy in the R-charges and dimensions leads to an exponential hierarchy in the suppression factors, and a generational structure in the Yukawa couplings automatically emerges even though none was present above $M_>$.

A superconformal fixed point also suppresses many soft SUSY-breaking terms. If a superconformal fixed point is stable, then all nonzero coupling constants approach fixed values in the infrared. A necessary condition for this to be true involves a certain property of the beta functions for all nonzero couplings in the vicinity of the fixed point; the matrix

$$\frac{\partial \beta_i}{\partial y_j} \tag{3.1}$$

must be positive definite, where β_i is the beta function for coupling y_i and the y_i are the nonzero couplings at the fixed point. The positivity of this matrix assures that the couplings flow toward, not away from, their fixed values in the infrared.

However, a superconformal fixed point must also have no SUSY-breaking parameters; the coupling constants, thought of as background superfields, may have nonzero lowest components, but cannot have nonzero higher components. For example, the coupling superfield $\mathbf{y} = y + \theta^2 A$, y a Yukawa coupling and A the corresponding A-term, must have fixed point value $y = y_*$, $A = 0$. This implies that stability requires not only that y flow to y_* but also that A flow to zero. Fortunately, when the couplings, beta functions and anomalous dimensions are all written as superfields, it becomes clear that these two conditions are the same; the positivity of the matrix (3.1) ensures *both* that y flows in the infrared to its fixed value *and* that A flows to zero. Said another way, the positivity of the matrix (3.1) should be interpreted as a *superfield* condition; when expanded in components, it shows that all SUSY-violating components of coupling constants flow to zero. This result is shown in detail in an extended appendix, generalizing results of [7, 8, 12, 13, 16].

Still, this is not enough to prove that *all* SUSY-breaking parameters flow to zero at a superconformal fixed point. Certain combinations of soft scalar masses-squared will be higher components of coupling superfields, but other linear combinations will generally not be. At the fixed point these orthogonal linear combinations generally do not flow at all. In particular, as explained in appendix B, if there is a non-R U(1) global symmetry in the theory, then the sum of the soft masses-squared weighted by their U(1) charges will not run at the fixed point [11].

In order to solve the supersymmetric flavor problem via renormalization, we must therefore consider models of fermion masses, along the lines of [1], in which the $\mathrm{U}(1)_R$ symmetry of the superconformal algebra is uniquely determined for the first two generations. In appendix B we argue that in such theories, the conformal dynamics will drive the soft masses of the lighter-generation sfermions toward zero.

The lighter-generation scalar masses do not quite reach zero, due to violations of conformal symmetry in the standard model sector. As shown in appendix C, each scalar mass-squared approaches a flavor-dependent value of order

$$\tilde{m}^2 \sim \frac{\alpha_3}{4\pi} M_3^2$$

where α_3 and M_3 are the QCD coupling constant and gluino mass, and all quantities are to be evaluated at the scale $M_<$. For a gluino mass of order 1 TeV, which means $M_3 \sim 400\,\mathrm{GeV}$ at the scale $M_<$, this gives $\tilde{m}^2 \sim 10^3\,\mathrm{GeV}^2 \ll M_{\mathrm{SUSY}}^2$.

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In short, a well-chosen conformal sector will generate a fermion mass hierarchy and reduce all the A-terms and soft scalar masses for the light generations (and part of the third) to small values. This generic result is the output which will serve as input for the SSM regime — assuming that we can escape into it from the conformal regime.

4. Escape from the conformal regime

We now briefly consider the escape from the conformal regime. There are only weak constraints on the scales $M_>$ and $M_<$. The ratio $M_>/M_<$ must be sufficiently large that the Yukawa couplings and soft masses are driven to small values. However, the ratio $M_</M_W$ must be sufficiently large that the SSM sector can do its work; the gaugino masses must have room to drive the scalar masses back to acceptably high values. These constraints are not difficult to satisfy; typically $M_> \sim 10^{14-19}$ GeV, $M_< \sim 10^{10-16}$ GeV can lead to reasonable models.

How can the theory depart the conformal regime at the scale $M_<$? The most natural possibility is that some nontrivial dynamics occurring at that scale leads the group G to disappear, perhaps through confinement. Another possibility is that some dynamics causes the standard model to decouple from the conformal sector at the scale $M_<$, with the conformal sector surviving to lower energies but coupling only irrelevantly to the standard model. In any case, the standard model fields X must decouple from that sector at a high scale $M_<$, and the sector itself (which contains some fields Q which are chiral and charged under S) must vanish well above the weak scale. Specific models of the escape were discussed in ref. [1]; suffice it to say that once the conformal sector is specified, it is neither easy nor impossible to construct such models.

On the other hand, the escape from the conformal regime must not ruin the flavor near-degeneracy that the conformal regime has worked so hard to build. In appendix D we will argue that in models with a rapid escape from a conformal regime to a weakly-coupled regime, the soft masses change only by factors of order one, and the small nonsupersymmetric mass terms for the first two generations of scalars remain small.

5. The flow in the supersymmetric standard model regime

After the escape, the low-energy theory consists merely of a supersymmetric version of the standard model; we will assume it is the minimal one. (Part of the conformal sector may still be around, but with only irrelevant couplings to the standard model sector.) The running of A-terms is controlled by the corresponding Yukawa couplings and by the A-terms themselves; for light fermions these are both small. The running of the gaugino masses and of the heavy-generation Yukawa couplings and A-terms, and of the masses of the corresponding squarks and possibly sleptons, are standard; these quantities may change by a factor of order one. We assume that the soft supersymmetry-breaking third-generation scalar masses are of the same order as the standard model gaugino masses, as necessary for a viable and natural model.[1] However, at $M_<$, the first two scalar generations are lighter by a factor of order $\sqrt{\frac{\alpha}{4\pi}}$, where α is a standard model gauge coupling.

After the conformal field theory decouples from the standard model, at the scale $M_<$, the soft parameters run according to the usual MSSM RG equations. We will consider them at leading loop order. The gaugino masses M_r, $r = 3, 2, 1$ for SU(3), SU(2) and U(1), run in the usual way, with M_r/α_r a one-loop renormalization-group invariant. The soft masses for the first two generations of sfermions are more interesting. Since for these generations the Yukawa couplings and A terms are small, there are only two important contributions at one loop, one from gaugino masses and one from the hypercharge D-term. These renormalizations are independent of the soft scalar masses themselves, and so are additive. Specifically, the mass-squared $\tilde{m}_s^2$ of the field ϕ_s has beta function

$$\beta_{\tilde{m}_s^2} = \sum_r \left| \frac{M_r}{\alpha_r} \right|^2 \frac{\partial^2 \gamma_s}{\partial(1/\alpha_r)^2} + \frac{1}{2\pi} \alpha_Y Y_s \tilde{m}_Y^2$$

where γ_s is the anomalous dimension of the field ϕ_s, Y_s is its hypercharge, and

$$\tilde{m}_Y^2 = \sum Y_j \tilde{m}_j^2$$

summed over all matter particles. (Here $\alpha_Y = (3/5)\alpha_1$, the latter being normalized to unify with the other couplings.) At one loop

$$\beta_{\tilde{m}_Y^2} = \frac{1}{2\pi} \left(\sum Y_j^2 \right) \alpha_1 \tilde{m}_Y^2 = -\frac{b_0^{(Y)}}{2\pi} \alpha_Y \tilde{m}_Y^2 = \frac{\beta_{\alpha_Y} \tilde{m}_Y^2}{\alpha_Y} \,.$$

Here $b_0^{(Y)}$ is the coefficient of the weak hypercharge one-loop beta function: $b_0^{(Y)} = -\sum_j Y_j^2$. Note that $\tilde{m}_Y^2/\alpha_Y$ is RG invariant. We also have

$$\gamma_s = \cdots - \sum_r \frac{1}{\pi} \alpha_r C_{sr} + \cdots$$

where C_{sr} is the quadratic Casimir ($\mathrm{Tr}(t^A t^B) = C_s \delta^{AB}$, $C_s = (N^2-1)/2N$ for SU(N) and q_s^2 for U(1)) of the field ϕ_s under the gauge group G_r, whence

$$\frac{\partial^2 \gamma_s}{\partial(1/\alpha)^2} = -\frac{2}{\pi} \alpha_r^3 C_{sr} \,.$$

But

$$\beta_{\ln \alpha} = -\beta_{\ln \alpha^{-1}} = -\alpha \beta_{1/\alpha} = -\frac{\alpha b_0}{2\pi} \Rightarrow \alpha = -\frac{2\pi \beta_{\ln \alpha}}{b_0}$$

and

$$\beta_{\alpha^2} = 2\alpha^2 \beta_{\ln \alpha} = -\frac{\alpha^3 b_0}{\pi} \Rightarrow \alpha^3 = -\frac{\pi \beta_{\alpha^2}}{b_0}$$

[1] Even though the MSSM gauge and top fields are not participating in the superconformal dynamics, we can imagine hypotheses under which this assumption could be unwarranted. For instance if the MSSM gauge and top Yukawa couplings are both strong at the fundamental scale and run down to perturbative values at lower energies, and if the soft terms are also present at the fundamental scale, then the soft terms might be renormalized by an order of magnitude or so, in unpredictable directions.

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so

$$\beta_{\tilde m_s^2} = \sum_r \left|\frac{M_r}{\alpha_r}\right|^2 \frac{2\beta_{\alpha_r^2}}{b_0} C_{sr} - \frac{Y_s}{b_0^{(Y)}} \beta_{\tilde m_Y^2}$$

which implies

$$\tilde m_s^2(\mu) - \tilde m_s^2(\mu_0) = \sum_r \left|\frac{M_r}{\alpha_r}\right|^2 \frac{2C_{sr}}{b_0^{(r)}} \left[\alpha_r(\mu)^2 - \alpha_r(\mu_0)^2\right] - \frac{Y_s}{b_0^{(Y)}} \frac{\tilde m_Y^2}{\alpha_1} - [\alpha_1(\mu) - \alpha_1(\mu_0)]$$

(recall that in models with SU(5) or greater unification, as well as in other models of supersymmetry breaking with $M_r \propto \alpha_r$, the factor M_r/α_r will be the same for the three standard model gauge groups.)

Thus the soft mass-squared parameters for the sfermions at the TeV scale take the form

$$\tilde m_s^2 = \sum_r \left|\frac{M_r}{\alpha_r}\right|^2 \frac{2C_{sr}}{b_0^{(r)}} \left[\alpha_r^2 - \alpha_{r<}^2\right] + \frac{Y_s}{b_0^{(Y)}} \frac{\tilde m_Y^2}{\alpha_1} [\alpha_{1<} - \alpha_1]\,; \tag{5.1}$$

plus a flavor-dependent piece of order $\alpha_{3<} M_{3<}^2/4\pi$. In this expression, quantities with a subscript $<$ are evaluated at $M_<$ while all others are evaluated at the scale of the sfermion masses.

We cannot measure $M_<$ very well. Consider $\tilde m_Y^2 = 0$. Since $\tilde m_{s<}^2$ is very small,

$$\left|\frac{\partial(\tilde m_s^2/M_3^2)}{\partial \ln M_<}\right| = \left|\frac{\beta_{\tilde m_s^2}\Big|_{\mu=M_<}}{|M_3|^2}\right| \approx \frac{\sum_r 2\alpha_{r<} M_{r<} C_{sr}}{\pi M_3^2} = \frac{\sum_r 2\alpha_{r<}^3 C_{sr}}{\pi \alpha_3^2}$$

which is of order 1/100. Therefore we cannot measure $\ln M_<$ from the soft masses to better than about 10. On the other hand, this means that the theory predicts certain combinations of squark and slepton masses.

For squarks the term involving α_3 is dominant, so the ratio of squark and gluino masses is

$$\frac{\tilde m_{\tilde q}^2}{M_3^2} \approx \frac{8}{9}\left[1 - \frac{\alpha_{3<}^2}{\alpha_3^2}\right].$$

Since this is not very sensitive to $\ln M_<$, and since $M_<$ cannot be taken too low, it is a prediction of the model that $\tilde m_{\tilde q}/|M_{\tilde g}| \sim .8-.9$. More detailed predictions will require going beyond one loop. We can also predict the splitting between the different types of quarks. The splitting between up-type and down-type SU(2)-singlet squark mass parameters is is

$$\tilde m_{\bar u}^2 - \tilde m_{\bar d}^2 \approx \frac{2}{33}\left[\left|\frac{M_1}{\alpha_1}\right|^2 \left[\alpha_{1<}^2 - \alpha_1^2\right] + \frac{3}{2}\frac{\tilde m_Y^2}{\alpha_1}(\alpha_{1<} - \alpha_1)\right], \tag{5.2}$$

while

$$\tilde m_q^2 - \frac{1}{2}\left(\tilde m_{\bar u}^2 + \tilde m_{\bar d}^2\right) \approx \frac{3}{2}\left|\frac{M_2}{\alpha_2}\right|^2 \left[\alpha_{2<}^2 - \alpha_2^2\right] - \frac{1}{22}\left|\frac{M_1}{\alpha_1}\right|^2 \left[\alpha_{1<}^2 - \alpha_1^2\right] - \frac{1}{33}\frac{\tilde m_Y^2}{\alpha_1}(\alpha_{1<} - \alpha_1)\,. \tag{5.3}$$

Note that both of these splittings can be larger than the expected small splittings at the scale $M_<$.

For sleptons, consider the sum $\tilde{m}_\nu^2 + \tilde{m}_{e^-}^2 + \tilde{m}_{e^+}^2$. Since the hypercharges of these particles cancel,

$$\frac{\tilde{m}_\nu^2 + \tilde{m}_{e^-}^2 + \tilde{m}_{e^+}^2}{3} = \left|\frac{M_2}{\alpha_2}\right|^2 [\alpha_{2<}^2 - \alpha_2^2] + \frac{1}{11}\left|\frac{M_1}{\alpha_1}\right|^2 [\alpha_{1<}^2 - \alpha_1^2]$$

Meanwhile,

$$\frac{1}{2}(\tilde{m}_\nu^2 + \tilde{m}_{e^-}^2) - \tilde{m}_{e^+}^2 \approx \frac{3}{2}\left|\frac{M_2}{\alpha_2}\right|^2 [\alpha_{2<}^2 - \alpha_2^2] - \frac{3}{22}\left|\frac{M_1}{\alpha_1}\right|^2 [\alpha_{1<}^2 - \alpha_1^2] + \frac{3}{22}\frac{\tilde{m}_Y^2}{\alpha_1}(\alpha_{1<} - \alpha_1)\,. \quad (5.4)$$

This splitting can easily be much larger than that inherited from the scale $M_<$. It thus serves to measure $\tilde{m}_Y^2$, leading to predictions for the splittings in the squark sector, eqs. (5.2) and (5.3).

Note that these formulas refer to soft supersymmetry-breaking mass terms. The physical scalar masses will receive tiny corrections from mixing and more important corrections from electroweak symmetry breaking via the quartic electroweak D-terms.

Unless R parity is violated, the splittings must be such that a neutral particle, either the lightest neutralino or the sneutrino, is the LSP. Our scenario does not constrain the μ term, so it is possible that the LSP is partly or altogether Higgsino; we will assume $\mu \gg M_1$ for the following estimate. Requiring that the right-handed selectron and smuon be heavier than the Bino implies

$$1 < \frac{\tilde{m}_{e^+}^2}{|M_1|^2} = \frac{1}{11}\left[2\left(\frac{\alpha_{1<}^2}{\alpha_1^2} - 1\right) - \frac{\tilde{m}_Y^2}{|M_1|^2}\left(\frac{\alpha_{1<}}{\alpha_1} - 1\right)\right] - 0.23\frac{m_Z^2}{|M_1|^2}\cos(2\beta)\,. \quad (5.5)$$

(Here we have written the physical mass of the sleptons, not merely their mass-squared parameter; the last term is due to the electroweak D-terms.) For $M_< = 10^{16}$ GeV, eq. (5.5) requires that either $M_1 < 120\,\mathrm{GeV}$ or $\tilde{m}_Y^2$ is negative. For $M_1 \gg M_Z$ we must have $|\tilde{m}_Y^2| > 1.3|M_1^2|$ for $M_< \sim M_{\mathrm{GUT}}$. Lower $M_<$ will require more negative $\tilde{m}_Y^2$. Large negative $\tilde{m}_Y^2$ could make the splitting in eq. (5.4) small, or negative, so the sneutrino could be the lightest scalar and even the LSP.

5.1 Examples

Here we give a couple of sample spectra, in order to demonstrate that there is a range of viable possibilities in which the $\tilde{e}^+$ is not the LSP. Because our scenario places no constraints on masses of third generation particles, we only present results for the first two generations. We use one-loop renormalization group equations, and neglect threshold corrections. All masses are renormalized at a scale of 500 GeV.

For our first example, we take

$$M_< = M_{\mathrm{GUT}} = 1.6 \cdot 10^{16}\,\mathrm{GeV}\,, \qquad \alpha_{\mathrm{GUT}} = 1/24.5\,, \qquad M_{1/2}(M_{\mathrm{GUT}}) = 400\,\mathrm{GeV}\,,$$
$$\text{D-term: } m_Y^2(M_{\mathrm{GUT}}) = -(300\,\mathrm{GeV})^2\,.$$

The Bino, Wino and gluino mass parameters at 500 GeV are

$$171, \; 332, \; 1015 \text{ GeV}$$

and the approximate masses for $\tilde{u}$, $\tilde{d}$, $\tilde{\bar{u}}$, $\tilde{\bar{d}}$, $\tilde{\nu}$, $\tilde{e}^-$, $\tilde{e}^+$ are

$$\begin{aligned}
&\left((922\,\text{GeV})^2 + 0.35 M_Z^2 \cos(2\beta)\right)^{\frac{1}{2}}, && \left((922\,\text{GeV})^2 - 0.42 M_Z^2 \cos(2\beta)\right)^{\frac{1}{2}}, \\
&\left((884\,\text{GeV})^2 + 0.15 M_Z^2 \cos(2\beta)\right)^{\frac{1}{2}}, && \left((882\,\text{GeV})^2 - 0.07 M_Z^2 \cos(2\beta)\right)^{\frac{1}{2}}, \\
&\left((278\,\text{GeV})^2 + 0.5 M_Z^2 \cos(2\beta)\right)^{\frac{1}{2}}, && \left((278\,\text{GeV})^2 - 0.27 m_Z^2 \cos(2\beta)\right)^{\frac{1}{2}}, \\
&\left((168\,\text{GeV})^2 - 0.23 M_Z^2 \cos(2\beta)\right)^{\frac{1}{2}}.
\end{aligned}$$

For $\tan\beta > 2$, the $\tilde{e}^+$ is not the LSP. Small, flavor non-degenerate, corrections to these masses come from the nonvanishing of the masses at the scale $M_<$. The natural size is

$$\Delta \tilde{m}^2 \sim \left.\frac{\alpha_3 M_3^2}{4\pi}\right|_{\mu = M_<} \sim 520\,\text{GeV}^2 \approx (23\,\text{GeV})^2 ,$$

a 0.06% effect for the squarks and a 2% effect for the sleptons. (Note the slepton splitting is dominated by α_3, since the order-one nonperturbative anomalous dimensions of lepton superfields are generated by fields in the conformal sector, which couple to SU(3)-color. These do not vanish in the limit $\alpha_1 \to 0$, $\alpha_2 \to 0$. See appendix C.)

If now we take a lower $M_<$, and continue to assume gauge coupling and gaugino mass unification at the usual GUT scale,[2] e.g.

$$\begin{aligned}
M_< = 1.6 \cdot 10^{12}\,\text{GeV}\,, \qquad (\alpha_1, \alpha_2, \alpha_3)(M_<) &= (1/34.2, \; 1/26.0, \; 1/20.1)\,, \\
(M_1, M_2, M_3)(M_<) &= (215, 283, 366)\,\text{GeV}\,, \\
\text{D-term:} \;\; m_Y^2(M_<) &= -(600\,\text{GeV})^2
\end{aligned}$$

(note the large $m_Y^2(M_<)$, needed to avoid an $\tilde{e}^+$ LSP) then the Bino, Wino and gluino mass parameters at 500 GeV are

$$129, \; 250, \; 762\,\text{GeV}\,.$$

Up to the aforementioned small nondegenerate corrections to the masses from the scale $M_<$, the masses for $\tilde{u}$, $\tilde{d}$, $\tilde{\bar{u}}$, $\tilde{\bar{d}}$, $\tilde{\nu}$, $\tilde{e}^-$, $\tilde{e}^+$ are

$$\begin{aligned}
&\left((653\,\text{GeV})^2 + 0.35 M_Z^2 \cos(2\beta)\right)^{\frac{1}{2}}, && \left((653\,\text{GeV})^2 - 0.42 M_Z^2 \cos(2\beta)\right)^{\frac{1}{2}}, \\
&\left((625\,\text{GeV})^2 + 0.15 M_Z^2 \cos(2\beta)\right)^{\frac{1}{2}}, && \left((634\,\text{GeV})^2 - 0.07 M_Z^2 \cos(2\beta)\right)^{\frac{1}{2}}, \\
&\left((146\,\text{GeV})^2 + 0.5 M_Z^2 \cos(2\beta)\right)^{\frac{1}{2}}, && \left((146\,\text{GeV})^2 - 0.27 m_Z^2 \cos(2\beta)\right)^{\frac{1}{2}}, \\
&\left((136\,\text{GeV})^2 - 0.23 M_Z^2 \cos(2\beta)\right)^{\frac{1}{2}}.
\end{aligned}$$

Note the near degeneracy between the left- and right-handed sleptons — in this case, for $\tan\beta > 1.7$, the sneutrino is the lightest scalar, although the Bino remains the LSP.

[2]Above the scale $M_<$, we cannot compute in a model independent way the effect of the strong dynamics on the individual β functions; however, provided that the strong dynamics respects an approximate global SU(5) symmetry, the gauge couplings and gaugino masses will still unify at 1.6×10^{16} GeV.

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As discussed in appendix C, the expected size of the flavor nondegeneracy in the mass-squared of all the scalars is again of order

$$\Delta\tilde{m}^2 \sim \frac{\alpha_3 M_3^2}{4\pi}\bigg|_{\mu=M_<} \approx (23\ \text{GeV})^2\,,$$

now an 2.6% effect for the sleptons and an 0.1% effect for the squarks.

6. Remnant flavor changing neutral currents

In the last section we computed the flavor-diagonal additive contribution that the light-generation scalar soft masses obtain through standard model loops and the gaugino masses. More precisely, the soft masses should be written as matrices $\tilde{m}^2_{ij}$ in generation space. In the basis where the MSSM fields have definite anomalous dimensions at the infrared fixed point, all matrix elements except $\tilde{m}^2_{33}$ will be small at $M_<$. However, we must estimate more carefully their size.

We will imagine that supersymmetry breaking occurs through some mechanism which generates gaugino masses of order $m_{1/2}$, scalar masses of order m_0, and A terms of order A_0. Either or both $m_{1/2}$ and A_0 could be naturally small, but we would not normally expect m_0 to be small. Let us assume initially, however, that all are of roughly the same order.

Assuming the strong dynamics is consistent with SU(5) symmetry, at $M_<$ the $\tilde{m}^2_{33}$ element of the $\tilde{q}, \tilde{\bar{u}}, \tilde{e}$ matrices for will be of order m_0^2. Unless $\tan\beta$ is very large, at $M_<$ all elements of $\tilde{\ell}$ and $\tilde{\bar{d}}$, including $\tilde{m}^2_{33}$, are small.

As previously noted and shown in appendix C, at $M_<$ diagonal mass matrix elements for scalar partners of light fermions have unknown values (positive or negative) of order $\eta \equiv \alpha_{3<} M^2_{3<}/(4\pi)$. In appendix C we also show that off-diagonal factors are of order

$$\epsilon_{ij} = \mathcal{O}\left(\left(\frac{M_<}{M_>}\right)^{(d_i+d_j-2)} m_0^2\right)\,,$$

where d_i is the scaling dimension of the appropriate field in the conformal regime. Therefore, after running from $M_<$ to low energy, the scalar mass-squared matrices take the form

$$\tilde{m}^2_{ij} \sim \begin{bmatrix} \tilde{m}_s^2 + \mathcal{O}\left(\eta m^2_{1/2}\right) & \epsilon_{12} & \epsilon_{13} \\ \epsilon_{12} & \tilde{m}_s^2 + \mathcal{O}\left(\eta m^2_{1/2}\right) & \epsilon_{23} \\ \epsilon_{13} & \epsilon_{23} & \tilde{m}^2_{33} \end{bmatrix}. \tag{6.1}$$

Here $\tilde{m}_s^2$ is the appropriate flavor-independent contribution for this charge of sfermion, as given in eq. (5.1). The element $\tilde{m}^2_{33}$ is of order m_0^2 for type I fields and is $\tilde{m}_s^2 + \mathcal{O}\left(\eta m^2_{1/2}\right)$ for type II fields, where type I fields are those whose third generation is weakly coupled to the conformal sector, and type II are all others. Type I includes at least $\tilde{t}, \tilde{b}, \tilde{\bar{t}}$, probably $\tilde{\tau}^+$, perhaps $\tilde{\bar{b}}$, $\tilde{\tau}^-$ and $\tilde{\nu}_\tau$.

The matrix in eq. (6.1) is given in the basis selected dynamically by the CFP. To study the resulting flavor-changing physics, it is much more convenient to work in the quark mass eigenstate basis. In this basis the off-diagonal elements are dominated by the residual $\mathcal{O}\left(\eta m_{1/2}^2\right)$ effects. For type I fields, we have

$$\tilde{m}_{ij}^2(I) \sim \begin{bmatrix} \tilde{m}_s^2 + \mathcal{O}\left(\eta m_{1/2}^2\right) & V_{12}\mathcal{O}\left(\eta m_{1/2}^2\right) & V_{13}\mathcal{O}(m_0^2) \\ V_{12}\mathcal{O}\left(\eta m_{1/2}^2\right) & \tilde{m}_s^2 + \mathcal{O}\left(\eta m_{1/2}^2\right) & V_{23}\mathcal{O}(m_0^2) \\ V_{13}\mathcal{O}(m_0^2) & V_{23}\mathcal{O}(m_0^2) & \mathcal{O}(m_0^2) \end{bmatrix}, \tag{6.2}$$

where V is the matrix which rotates the CFP basis to the quark mass basis; its off-diagonal entries are of order

$$V_{ij} = \mathcal{O}\left(\frac{M_<}{M_>}\right)^{|d_i - d_j|} . \tag{6.3}$$

For type II fields, the off-diagonal entries involving the third generation are smaller, and in the quark mass eigenstate basis we have

$$\tilde{m}_{ij}^2(II) \sim \begin{bmatrix} \tilde{m}_s^2 + \mathcal{O}\left(\eta m_{1/2}^2\right) & V_{12}\mathcal{O}\left(\eta m_{1/2}^2\right) & V_{13}\mathcal{O}\left(\eta m_{1/2}^2\right) \\ V_{12}\mathcal{O}\left(\eta m_{1/2}^2\right) & \tilde{m}_s^2 + \mathcal{O}\left(\eta m_{1/2}^2\right) & V_{23}\mathcal{O}\left(\eta m_{1/2}^2\right) \\ V_{13}\mathcal{O}\left(\eta m_{1/2}^2\right) & V_{23}\mathcal{O}\left(\eta m_{1/2}^2\right) & \tilde{m}_s^2 + \mathcal{O}\left(\eta m_{1/2}^2\right) \end{bmatrix}, \tag{6.4}$$

The off-diagonal terms in these matrices will induce flavor-changing neutral currents (FCNCs) and violations of CP, and we must check that the models under discussion are consistent with experimental constraints.

We also have FCNCs and CP violation from the A-terms. These inherit roughly the same structure as the Yukawa couplings. If unstructured at the scale $M_>$, at low energy they are of the form

$$A_{ij} \sim (\mathrm{Max}(y_i, y_j)) \left(\mathrm{Max}\left(V_{ij}^L, V_{ij}^R\right)\right) A_0 \,, \tag{6.5}$$

where and $V_{ij}^{L,R}$ refer to the left- and right-handed fields respectively. For the quark fields, $V^L \sim V^{CKM}$. We expect A_0 to be of order m_0 or smaller, depending in detail on the supersymmetry breaking and communication mechanism.

In determining the constraints on our scenario, we use the review of Gabbiani et al. [17], supplemented as necessary to incorporate QCD corrections [18]–[24]. We will see that this mechanism for flavor changing neutral current suppression is roughly consistent with the constraints, although adequate suppression of $\mu \to e\gamma$ and the electron electric dipole moment constrains A_0 to be significantly smaller than m_0.

6.1 K-$\bar{K}$ system

The K_L-K_S mass difference and ϵ_K place constraints on the down-type squark masses. For our scenario, the strongest constraints are

$$\left|\mathrm{Re}(\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR}\right|^{1/2} < 1.2 \times 10^{-3}\left(\frac{m_{\tilde{d}}}{500\,\mathrm{GeV}}\right) \tag{6.6}$$

$$\left|\mathrm{Im}(\delta_{12}^d)_{LL}(\delta_{12}^d)_{RR}\right|^{1/2} < 0.1 \times 10^{-3}\left(\frac{m_{\tilde{d}}}{500\,\mathrm{GeV}}\right). \tag{6.7}$$

where,$(\delta^d_{12})_{LL(RR)}$ is the ratio of the off-diagonal term to the diagonal terms in the left-(right)-handed squark mass-squared matrix, in the basis (see above) which diagonalizes the down-type quark masses. We assume the left- and right-handed $\tilde{m}^2_{12}$ are roughly of the same order of magnitude — otherwise the constraints are much weaker. The expected non-degeneracy in the squark masses is of order $\alpha_s/(4\pi) \sim 10^{-3}$, and the off-diagonal terms should additionally be suppressed by mixing angles — here we assume both left- and right-handed mixing angles to be of order the Cabbibo angle. Then both $(\delta^d_{12})_{LL}$ and $(\delta^d_{12})_{RR}$ are expected to be of order 0.2×10^{-3}, potentially saturating the constraints coming from CP violation.

Indirect evidence for a sizable supersymmetric contribution to ϵ_K might arise from a B-factory measurement of the CP asymmetry in $B_d(\bar{B}_d) \to \psi K_S$. Should this asymmetry disagree with the standard model prediction, one explanation would be that the CKM phase cannot be deduced from ϵ_K using the standard model alone.

Much less constraining are the flavor changing terms mixing the left- and right-handed squarks, since these, which come from the A-terms, are always suppressed by a factor of order the strange quark Yukawa coupling times the Cabbibo angle.

$$\left|(\delta^d_{12})_{LR}(\delta^d_{12})_{RL}\right|^{1/2} \sim \frac{y_s}{\sqrt{4\pi}}\theta_C < 2.4 \times 10^{-3}\left(\frac{m_{\tilde{d}}}{500\,\mathrm{GeV}}\right).$$

6.2 B_d-$\bar{B}_d$ mixing

Again, for B_d-$\bar{B}_d$ mixing, the strongest constraints are on the product of the off-diagonal terms in the product of the left- and right-handed squark mixing matrices.

$$\left|(\delta^d_{13})_{LL}(\delta^d_{13})_{RR}\right|^{1/2} < 2 \times 10^{-2}\left(\frac{m_{\tilde{d}}}{500\,\mathrm{GeV}}\right). \tag{6.8}$$

While the nondegeneracy in this system may be of order 1, the $\tilde{m}^2_{13}$ off-diagonal terms in the left-handed squark mass matrices will be suppressed by a factor of order $V_{td} \sim 10^{-2}$. In the right-handed matrices there are two possible cases. The $\bar{b}$ could be a type I field, in which case we expect

$$(\delta^d_{13})_{RR} \sim V^R_{13} \sim \frac{m_d/m_b}{V_{td}}$$

and

$$\sqrt{|(\delta^d_{13})_{LL}(\delta^d_{13})_{RR}|}(I) \sim \sqrt{\frac{m_d}{m_b}} \sim 3 \times 10^{-2}$$

If the $\bar{b}$ is a type II field, then V^R_{13} is also suppressed by a factor of $\eta \sim 10^{-3}$.

In both cases constraint (6.8) is easily satisfied. For type I right handed b quark, and for 600 GeV or lighter squarks, there is a possible $\mathcal{O}(1)$ contribution from supersymmetry to the phase of $B_d - \bar{B}_d$ mixing, which could show up as an non-standard value for the time dependent CP asymmetries in $B_d(\bar{B}_d) \to f_{CP}$ decays.

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6.3 B_s-$\bar{B}_s$ mixing

In the standard model the ratio of the B_s mixing to B_d mixing is predicted to give approximately

$$\left.\frac{\Delta m_{B_s}}{\Delta m_{B_d}}\right|_{\text{standard}} \approx \left|\frac{V_{ts}^{CKM}}{V_{td}^{CKM}}\right|^2 \sim 20\,.$$

This prediction should be tested at upcoming measurements of the B_s oscillation rate at the Tevatron. It is therefore interesting to compare the ratio of the flavor-violating supersymmetric contributions to the mixing, which is of order

$$\frac{(\delta_{23}^d)_{LL}(\delta_{23}^d)_{RR}}{(\delta_{13}^d)_{LL}(\delta_{13}^d)_{RR}} \sim \frac{V_{23}^d V_{23}^{\bar{d}}}{V_{13}^d V_{13}^{\bar{d}}} \sim \frac{m_s}{m_d} \sim 20\,.$$

We therefore do not expect a big correction to the standard model prediction for the ratio. The phase in the supersymmetric contribution to B_s-$\bar{B}_s$ mixing could well be nonstandard, however, and lead to an $\mathcal{O}(1)$ CP-violating asymmetry in, e.g. time dependent $B_s(\bar{B}_s) \to (J/\psi)\eta$, which would be a clear signal of beyond the standard model physics.

6.4 Constraints on A-terms from $\epsilon'/\epsilon, \mu \to e\gamma$, and electric dipole moments

Our model falls into the class of models with A-terms of the same texture as the Yukawa couplings, which were recently analyzed by Masiero and Murayama [15]. They found that for 500 GeV sleptons, CP-violating A terms lead to an electron electric dipole moment (EDM) of size comparable to the current bounds, and lepton flavor-violating A terms lead to a $\mu \to e\gamma$ rate of order the experimental bounds. Note that with gaugino mass unification our scenario leads to sleptons which are much lighter than the gluino and squarks. Since natural electroweak symmetry breaking places stringent upper bounds on the gluino mass [25]–[28] we expect rather light sleptons. For a gluino lighter than 1 TeV the left-handed slepton mass is lighter than 300 GeV.

Lepton flavor violation places a constraint

$$|(\delta_{12}^\ell)_{LR}| < 1.7 \times 10^{-6} \left(\frac{\tilde{m}_\ell}{100\,\text{GeV}}\right)^2 ,$$

whereas, using eq. (6.5), our scenario gives

$$|(\delta_{12}^\ell)_{LR}| \sim \frac{A_0 m_\mu \,\text{Max}(V_{12}^{\ell^-}, V_{12}^{\ell^+})}{\tilde{m}_\ell^2} \sim 8 \times 10^{-5} \left(\frac{A_0}{100\,\text{GeV}}\right)\left(\frac{(100\,\text{GeV})^2}{\tilde{m}_\ell^2}\right),$$

where in the last step we have assumed

$$\text{Max}\left(V_{12}^{\ell^-}, V_{12}^{\ell^+}\right) \sim \sqrt{\frac{m_e}{m_\mu}}\,.$$

We therefore must require that

$$\frac{A_0}{100\,\text{GeV}} \lesssim 0.02 \left(\frac{\tilde{m}_\ell}{100\,\text{GeV}}\right)^4 . \qquad (6.9)$$

The constraint from the electron EDM is

$$\mathrm{Im}(\delta^{\ell}_{11})_{LR} < 3.7 \times 10^{-7} \left(\frac{\tilde{m}_{\ell}}{100\,\mathrm{GeV}}\right)^2 , \tag{6.10}$$

whereas we expect

$$|(\delta^{\ell}_{11})_{LR}| \sim \frac{A_0 m_e}{\tilde{m}^2_{\ell}} \sim 5 \times 10^{-6} \left(\frac{A_0}{100\,\mathrm{GeV}}\right) \left(\frac{(100\,\mathrm{GeV})^2}{\tilde{m}^2_{\ell}}\right) .$$

Thus the electron EDM places a slightly less stringent constraint on A_0 than $\mu \to e\gamma$. The constraint from the neutron EDM will be even weaker, since the squarks are so much heavier than the sleptons. Similarly, one can look at constraints on A terms from $\tau \to \mu\gamma$. We have the constraint

$$|(\delta^{\ell}_{23})_{LR}| < 2.0 \times 10^{-2} \left(\frac{\tilde{m}_{\ell}}{100\,\mathrm{GeV}}\right)^2 , \tag{6.11}$$

whereas we from eq. (6.5) we expect

$$(\delta^{\ell}_{23})_{LR} \sim \frac{A_0 m_{\tau}}{\tilde{m}^2_{\ell}} \sim 1.7 \times 10^{-2} \left(\frac{A_0}{100\,\mathrm{GeV}}\right) \left(\frac{(100\,\mathrm{GeV})^2}{\tilde{m}^2_{\ell}}\right) , \tag{6.12}$$

where we have assumed maximal mixing between the left-handed τ and μ. This constraint on A_0 is also less severe.

We conclude that in this scenario the natural expectations for EDM's and the $\mu \to e\gamma$ rate are somewhat larger than the experimental bounds, unless there is some additional suppression of A terms relative to gaugino masses.

For squark masses of 500 GeV there is a supersymmetric contribution to ϵ'/ϵ of order the experimental size from graphs containing CP- and flavor-violating A terms. Since the squarks and gluino must be rather heavy, and A_0 small, the expectation for supersymmetric contributions to ϵ'/ϵ is below the experimental value. Notice that we avoid the significant supersymmetric contribution to ϵ'/ϵ discussed in ref. [29], since $\tilde{\tilde{u}}$ and $\tilde{\tilde{d}}$ squarks are degenerate to a few percent.

6.5 $b \to s\gamma$

The constraints from $b \to s\gamma$ are not simple since there may be several supersymmetric contributions, even for flavor-degenerate scalar mass matrices. However, roughly speaking we must have

$$|(\delta^{d}_{23})_{LR}| \sim \frac{y_b}{\sqrt{4\pi}} V_{cb} < 10^{-2} \left(\frac{m_{\tilde{d}}}{500\,\mathrm{GeV}}\right)^2$$

This is not a serious concern.

7. Summary

We have shown that both the flavor hierarchy and the absence of unacceptable flavor changing neutral currents from superpartner exchange may be explained by couplings of the first two generations of quark and lepton superfields to a superconformal sector at short distances. Using some of the recent exact results on renormalization of soft supersymmetry breaking terms, we find a distinctive imprint on the superpartner spectrum. The masses of the superpartners of the first two generations are flavor-degenerate, and related to the gaugino masses, and the trilinear scalar terms must be relatively small, although they will inherit the basic flavor structure of the Yukawa couplings. There is almost complete freedom for the other soft supersymmetry breaking parameters.

While this work was in the process of completion reference [30] appeared, which contains some overlapping results.

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A. Generalization of exact formulas for running of soft supersymmetric terms

In this section, we discuss some exact formulas for running of supersymmetry-preserving and -breaking couplings. All of this material has appeared in the literature [2, 4, 3, 5, 6], [9]–[12]; however, it has not been well-summarized elsewhere.

As a warm-up, consider a theory with four supercharges in two, three or four dimensions. The theory has a single chiral superfield ϕ and lagrangian

$$\int d^4\theta\phi^\dagger\phi + \left[\int d^2\theta\ W(\phi) + \text{h.c.}\right]$$

where the superpotential W takes the form

$$W(\phi) = \hat{y}_2\phi^2 + \hat{y}_3\phi^3 + \hat{y}_4\phi^4 + \cdots$$

For the moment, we limit ourselves to perturbatively renormalizable models, implying $\hat{y}_k = 0$ for all $k > 4$ in $d = 3$ and for all $k > 3$ in $d = 4$.

We have defined the theory, and normalized the kinetic term of the scalars, at a certain scale μ_0. At this scale the physical couplings y_k are the same as the holomorphic couplings $\hat{y}_k$. However, if we consider the theory at a lower scale μ, using a Wilsonian prescription, the same theory must be written

$$\int d^4\theta Z(\mu)\phi^\dagger\phi + \left[\int d^2\theta\ (\hat{y}_2\phi^2 + \hat{y}_3\phi^3 + \hat{y}_4\phi^4 + \cdots) + \text{h.c.}\right]$$

where the absence of any vertex factors is a consequence of the non-renormalization theorems. Now the physical coupling is related to the holomorphic one by

$$y_k(\mu) = Z(\mu)^{-k/2}\hat{y}_k \equiv \sqrt{Z_{y_k}(\mu)}\hat{y}_k\,.$$

From this we see

$$\beta_{y_k} = \frac{\partial y_k}{\partial \ln \mu} = \frac{k}{2} y_k \gamma_\phi \equiv \frac{1}{2} y_k \gamma_{y_k} \,, \qquad \text{i.e.,} \quad \beta_{\ln y_k} = \frac{1}{2}\gamma_{y_k}$$

where

$$\gamma_\phi = -\frac{\partial \ln Z}{\partial \ln \mu}$$

is the anomalous mass dimension of the field ϕ (positive, by unitarity, for all gauge invariant fields), and

$$\gamma_{y_k} = -\frac{\partial \ln Z_{y_k}}{\partial \ln \mu} = k\gamma_\phi$$

measures the dimensionality of the coupling y_k.

Since y_k has canonical dimension $d^0(y_k) = (d-1) - kd_\phi = d - 1 - k(d-2)/2$, its beta function does not vanish at a conformal fixed point; instead, a fixed point has

$$\beta_{y_k} = d^0(y_k) \tag{A.1}$$

It is often useful instead to define a dimensionless coupling

$$\mathcal{Y}_k = y_k \mu^{k(\frac{d}{2}-1)+1-d}$$

with

$$\beta_{\ln \mathcal{Y}_k} = \frac{\partial \ln \mathcal{Y}_k}{\partial \ln \mu} = \frac{1}{2}\left[k(d-2) - 2d + 2 + k\gamma_\phi\right] \equiv \frac{1}{2}\gamma_{\mathcal{Y}_k}$$

so that a fixed point of a theory occurs at $\beta_{\mathcal{Y}_k} = 0$. However, we will not usually need this.

Note that these beta functions are not generally related to the presence of short distance singularities. For example, in three dimensions the theory

$$W = y_3 \phi^3$$

is finite; but the beta function is non-zero and reaches a fixed point in the infrared.

Note also that it is essential to treat mass thresholds in a particular way in order to maintain this structure through the threshold.

A.1 A simple example

Now let us consider the case where only y_3 is non-zero; in the following we will drop the subscript 3. The beta function is

$$\beta_y = \frac{1}{2} y[3\gamma_\phi] = \frac{1}{2} y \gamma_y \,.$$

Thus the theory reaches a fixed point $\gamma_\phi = (4-d)/3$. There can be no such fixed point in four dimensions (since γ must be non-zero and positive for an interacting gauge-invariant field) but a fixed point exists for any dimension less than four. (This is the supersymmetric Wilson-Fisher fixed point.)

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Next, let us consider the effect of supersymmetry breaking in this theory. At the cutoff scale, we may add a trilinear scalar term $A_0\phi^3$ and a scalar mass term $\tilde{m}_0^2\phi^*\phi$. We can implement the first term by making y a superfield of the form $\mathbf{\hat{y}} = \hat{y}+\theta^2\hat{A}$. It is convenient to write

$$\mathbf{\hat{y}} = \hat{y}e^{\theta^2(\hat{A}/\hat{y})}\,.$$

The scalar mass term may be added by making Z a superfield of the form $\mathbf{Z} = Z(1-\theta^4\tilde{m}_0^2)$ at some scale μ_0. However, while $\mathbf{\hat{y}}$ is holomorphic and is not renormalized, the same is not true for $\mathbf{Z}$. At any $\mu \neq \mu_0$, we must assume it is a general real superfield with θ^2, $\bar{\theta}^2$ and $\theta^4 = \theta^2\bar{\theta}^2$ components. It is convenient to write

$$\mathbf{Z} = Ze^{\theta^2 C+\bar{\theta}^2 C^* -\theta^4\tilde{D}}$$

where $C = 0$ and $\tilde{D} = \tilde{m}_0^2$ at $\mu = \mu_0$.

Now, the theory has a global symmetry $\mathrm{U}(1)_\phi$ under which

$$\phi \to \phi e^{\mathbf{T}}\,, \qquad \phi^\dagger \to \phi^\dagger e^{\mathbf{T}^\dagger}\,, \qquad \mathbf{\hat{y}} \to \mathbf{\hat{y}}e^{-3\mathbf{T}}\,, \qquad \mathbf{Z} \to e^{-(\mathbf{T}^\dagger+\mathbf{T})}\mathbf{Z}\,. \tag{A.2}$$

where $\mathbf{T}$ is an arbitrary chiral superfield. Note that this symmetry can be used to change the scale μ_0 at which $C = 0$ and $\tilde{D} = \tilde{m}_0^2$; however the transformations are μ-independent and therefore cannot fix $C = 0$ at all μ. This symmetry is just a reparametrization of our theory, and physical observables must be invariant under such transformations. The theory also has an R symmetry under which ϕ and Z are invariant and $\mathbf{\hat{y}}$ transforms with charge 2. Let us construct some objects which are invariant under these symmetries. The simplest invariant superfield is

$$-\bar{D}^2D^2\ \ln\mathbf{Z} = \tilde{D}$$

which is independent of θ. Thus $\ln\mathbf{Z}|_{\theta^4}$ is a physically meaningful quantity, while $\ln\mathbf{Z}|_{\theta^2}$ is not. Another important invariant is the superfield

$$e^{\mathbf{H}_y} \equiv \mathbf{\hat{y}}^\dagger\mathbf{Z}^{-3}\mathbf{\hat{y}} = Z^{-3}|\hat{y}|^2e^{\theta^2\left(\frac{\hat{A}}{\hat{y}}-3C\right)+\bar{\theta}^2\left(\frac{\hat{A}^*}{\hat{y}^*}-3C\right)+\theta^4\tilde{D}_y}\,;$$

here $\tilde{D}_y \equiv 3\tilde{D}$.

These two invariants, $\tilde{m}^2$ and $\mathbf{H}_y$, completely specify the running coupling constants of the theory. (In this case, the D-term $\tilde{m}_y^2$ of $\mathbf{H}_y$ also specifies $\tilde{m}^2$. In some cases, however, there are independent invariants of the form $\bar{D}^2D^2\ \ln\mathbf{Z}$.) Let us define these physical parameters. At any scale μ_0, we may canonically normalize ϕ by making a global *non-holomorphic* version of the transformation (A.2), with $\mathbf{T} = \frac{1}{2}\ln Z + \theta^2 C$. After this transformation we have

$$\mathbf{\hat{y}} \to Z^{-3/2}\hat{y}e^{\left(\frac{\hat{A}}{\hat{y}}-3C\right)\theta^2}\,, \qquad \mathbf{Z} = e^{-\theta^4\tilde{D}}$$

from which we conclude that the physical Yukawa coupling is

$$y = Z^{-3/2}\hat{y}\,, \qquad \mathcal{Y} = y\mu^{d-4}$$

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(as in the supersymmetric case), the physical A-term is

$$A = Z^{-3/2}\hat{A} - 3C\hat{y} \Rightarrow \frac{A}{y} = \frac{\hat{A}}{\hat{y}} - 3C = \mathbf{H}_y|_{\theta^2}\,,$$

and the physical soft mass is

$$\tilde{m}^2 = -\bar{D}^2 D^2 \ln \mathbf{Z} = \tilde{D}\,.$$

It is useful to define $\tilde{m}_y^2 \equiv \tilde{D}_y = \mathbf{H}_y|_{\theta^4} = 3\tilde{m}^2$.

Now, this language makes the beta functions for the physical parameters extremely simple. Since

$$\mathbf{H}_y = \ln \hat{\mathbf{y}}^\dagger + \ln \hat{\mathbf{y}} - 3\ln \mathbf{Z}$$

and since $\hat{\mathbf{y}}$ is μ-independent by the non-renormalization theorem of $\mathcal{N}=1$ supersymmetry, we have a natural superfield beta function

$$\mathcal{B}_{\mathbf{H}_y} \equiv \frac{\partial \mathbf{H}_y}{\partial \ln \mu} = -3\frac{\partial \ln \mathbf{Z}}{\partial \ln \mu} \equiv -\frac{\partial \ln \mathbf{Z}_y}{\partial \ln \mu} \equiv \Gamma_y\,. \tag{A.3}$$

where Γ_y is a superfield anomalous dimension. Since $\mathbf{H}_y$ is invariant under the symmetries, the same must be true for Γ_y; this follows from the fact that under (A.2), $\ln \mathbf{Z}$ shifts additively by $\mathbf{T}+\mathbf{T}^\dagger$, which is μ-independent. It follows then that Γ_y is a function only of $\mathbf{H}_y$, $\bar{D}^2D^2\ln\mathbf{Z}$, and μ. However, since $\bar{D}^2D^2\ln\mathbf{Z}$ vanishes in a supersymmetric theory, and since supersymmetric formulas must be accurate when the scale of supersymmetry breaking is small compared to the scale μ, Γ_y can only depend on positive powers of $(\bar{D}^2D^2\ln\mathbf{Z})/\mu^2$. For μ much larger than all supersymmetry-breaking mass scales, the dependence on this linear combination can be neglected. In this paper, we need formulas appropriate for $M_{pl} > \mu > M_< \gg M_W$, and so we may take Γ_y to be a function of $\mathbf{H}_y$ only.

All θ^2 dependence therefore enters through $\mathbf{H}_y$. The $\theta = 0$ component of eq. (A.3) gives

$$\beta_{\ln|y|^2} = \gamma_y = 2\beta_{\ln y}\,,$$

which agrees as it must with the supersymmetric case. The θ^2 component of eq. (A.3) reads

$$\beta_{\frac{A}{y}} = \frac{\partial\gamma_y}{\partial \ln \mathbf{H}_y}\ln \mathbf{H}_y\Big|_{\theta^2} = \frac{A}{y}\frac{\partial \gamma_y}{\partial \ln|y|^2}\,,$$

while the θ^4 component gives

$$\begin{aligned}\beta_{\tilde{m}_y^2} &= 3\left[\frac{\partial^2\gamma_y}{\partial[\ln|y|^2]^2}\left|\left(\ln \mathbf{H}_y\Big|_{\theta^2}\right)\right|^2 + \frac{\partial\gamma_y}{\partial\ln|y|^2}\ln\mathbf{H}_y\Big|_{\theta^4}\right] = \\ &= \left[\left|\frac{A}{y}\right|^2\frac{\partial^2}{\partial[\ln|y|^2]^2} + 3\tilde{m}^2\frac{\partial}{\partial\ln|y|^2}\right]\gamma_y\,.\end{aligned}$$

More conventionally, we may use

$$\frac{\partial f(|y|^2)}{\partial \ln y} = \frac{\partial f(|y|^2)}{\partial \ln y^*} = \frac{\partial f(|y|^2)}{\partial \ln |y|^2}$$

and write

$$\beta_{\ln y} = \frac{1}{2}\gamma_y\,,$$
$$\beta_{\frac{A}{y}} = \frac{A}{y}\frac{\partial}{\partial \ln y}\gamma_y \equiv -D_1\gamma_y\,,$$

and

$$\beta_{\tilde{m}^2_y} = \left[\left|\frac{A}{y}\right|^2 \frac{\partial^2}{\partial \ln y\,\partial \ln y^*} + \tilde{m}^2_y \frac{\partial}{\partial \ln y}\right]\gamma_\phi \equiv \left[D_1^* D_1 + D_2\right]\gamma_y\,,$$

or equivalently, dividing by 3,

$$\beta_{\tilde{m}^2} = \left[D_1^* D_1 + D_2\right]\gamma_\phi\,.$$

where

$$D_1 \equiv -\frac{A}{y}\frac{\partial}{\partial \ln y}\,, \qquad D_2 \equiv \tilde{m}^2_y \frac{\partial}{\partial \ln y}\,.$$

Note that our formal definition of $D_1^* D_1$ requires that derivatives with respect to $\ln y^*$ do not act on A/y.

A.2 More couplings

Still working with one field ϕ, let us allow N_y of the renormalizable $\hat{y}_k$ to be non-zero. Let us call N_I the number of independent invariants $\mathbf{H}_m$. It is easy to see that there are many more invariants than there are couplings, since $\hat{\mathbf{y}}_k^\dagger \mathbf{Z}^{-2k}\hat{\mathbf{y}}_k \equiv \exp \mathbf{H}_k$ is always present, but for example $\hat{\mathbf{y}}_4^\dagger \hat{\mathbf{y}}_5^\dagger \mathbf{Z}^{-9}\hat{\mathbf{y}}_3\hat{\mathbf{y}}_6$ is an additional invariant under $\mathrm{U}(1)_\phi$ and $\mathrm{U}(1)_R$. In general, every coupling invariant will have the form

$$\mathbf{H}_m = \sum_k \mathcal{Q}_{mk} \ln \hat{\mathbf{y}}_k + \sum_k \bar{\mathcal{Q}}_{mk} \ln \hat{\mathbf{y}}_k^\dagger - \left(\sum_k \mathcal{Q}_{mk}\right) \ln \mathbf{Z} \tag{A.4}$$

where $\mathcal{Q}_{mk}$ and $\bar{\mathcal{Q}}_{mk}$ are two independent $N_I \times N_y$ matrices, with all entries positive or zero, satisfying the two constraints

$$\sum_k \mathcal{Q}_{mk} = \sum_k \bar{\mathcal{Q}}_{mk} \quad (\mathrm{U}(1)_R \text{ conservation}), \tag{A.5}$$

$$\sum_k k\mathcal{Q}_{mk} = \sum_k k\bar{\mathcal{Q}}_{mk} \quad (\mathrm{U}(1)_\phi \text{ conservation}). \tag{A.6}$$

The components of eq. (A.4) are, in terms of the physical parameters,

$$\mathbf{H}_m\Big|_{\theta=0} = \sum_k \mathcal{Q}_{mk} \ln y_k + \sum_k \bar{\mathcal{Q}}_{mk} \ln y_k^*$$
$$\mathbf{H}_m\Big|_{\theta^2} = \sum_k \mathcal{Q}_{mk}\frac{A_k}{y_k}\,, \qquad \mathbf{H}_m\Big|_{\bar\theta^2} = \sum_k \bar{\mathcal{Q}}_{mk}\frac{A_k^*}{y_k^*} \neq \left(\mathbf{H}_m\Big|_{\theta^2}\right)^*$$
$$\mathbf{H}_m\Big|_{\theta^4} = \sum_k \mathcal{Q}_{mk}\tilde{m}_k^2 = \sum_k \bar{\mathcal{Q}}_{mk}\tilde{m}_k^2$$

where the fact that $\tilde{m}_k^2 = k\tilde{m}^2$, combined with the $\mathrm{U}(1)_\phi$ constraint eq. (A.6), implies the two expressions for the θ^4 component are equal. From the components of $\mathbf{H}_k$, $k = 1, \ldots, N_y$, we confirm that the beta functions for the couplings y_k satisfy

$$\beta_{\ln y_k} = \frac{1}{2}\gamma_k\,, \qquad \beta_{\frac{A_k}{y_k}} = -D_1\gamma_k\,, \qquad \beta_{\frac{A_k^*}{y_k^*}} = -\bar{D}_1\gamma_k\,, \qquad \beta_{\tilde{m}_k^2} = (\bar{D}_1 D_1 + D_2)\gamma_k$$

where $\gamma_k = k\gamma_\phi$,

$$D_1 = -\sum_m \mathbf{H}_m\Big|_{\theta^2}\frac{\partial}{\partial\mathbf{H}_m}\,, \qquad \bar{D}_1 = -\sum_m \mathbf{H}_m\Big|_{\bar\theta^2}\frac{\partial}{\partial\mathbf{H}_m}\,,$$

(note $\bar{D}_1$ is not apparently the complex conjugate of D_1, since not all $\mathbf{H}_m$ are real) and

$$D_2 = \sum_m \mathbf{H}_m\Big|_{\theta^4}\frac{\partial}{\partial\mathbf{H}_m}\,.$$

But since, acting on any function of the invariants,

$$\frac{\partial}{\partial\ln y_k} = \sum_m \mathcal{Q}_{mk}\frac{\partial}{\partial\mathbf{H}_m}\,, \qquad \frac{\partial}{\partial\ln y_k^*} = \sum_m \bar{\mathcal{Q}}_{mk}\frac{\partial}{\partial\mathbf{H}_m}\,,$$

we have

$$\begin{aligned} D_1 &= -\sum_{k,m}\frac{A_k}{y_k}\mathcal{Q}_{mk}\frac{\partial}{\partial\mathbf{H}_m} = -\sum_k \frac{A_k}{y_k}\frac{\partial}{\partial\ln y_k}\,, \\ \bar{D}_1 &= -\sum_m \mathbf{H}_m\Big|_{\bar\theta^2}\frac{\partial}{\partial\mathbf{H}_m} = -\sum_j \frac{A_j^*}{y_j^*}\frac{\partial}{\partial\ln y_j^*} = (D_1)^*\,, \end{aligned}$$

and

$$\sum_m \mathbf{H}_m\Big|_{\theta^4}\frac{\partial}{\partial\mathbf{H}_m} = \sum_k \tilde{m}_k^2\frac{\partial}{\partial\ln y_k} = \sum_k \tilde{m}_k^2\frac{\partial}{\partial\ln y_k^*}$$

which implies

$$D_2 = \sum_j \tilde{m}_j^2\frac{\partial}{\partial\ln y_j} = \frac{1}{2}\tilde{m}_j^2\sum_j\left(\frac{\partial}{\partial\ln y_j} + \frac{\partial}{\partial\ln y_j^*}\right)\,.$$

So in the end we need not worry about the particular invariants in a given theory; D_1 and D_2 only involve derivatives with respect to the N_y physical couplings.

A.3 More fields

At this point it is straightforward to generalize to a theory with more fields ϕ_i with superpotential

$$W = \sum_{s=1}^{N_y} \hat{\mathbf{y}}_s \prod_i (\phi_i)^{\mathcal{P}_s^i}\,, \tag{A.7}$$

where $\mathcal{P}_s^i$ is a matrix with non-negative integer entries, and $\hat{\mathbf{y}}_s = \hat{y}_s + \theta^2\hat{A}_s$. If we assume that symmetries prevent mixing among the different ϕ's, there are N_y invariants of the form

$$e^{\mathbf{H}_s} = \hat{\mathbf{y}}_s^\dagger\hat{\mathbf{y}}_s\left(\prod_i \mathbf{Z}_i^{\mathcal{P}_s^i}\right)^{-1} \qquad (s = 1, \ldots, N_y)$$

where $\mathbf{Z}_i$ is the wave-function renormalization of ϕ_i. In addition there are invariants $\mathbf{H}_m$, $m = N_y + 1, \ldots, N_I$. As before, we have

$$\beta_{\ln y_s} = \frac{1}{2}\gamma_s \,, \qquad \beta_{\frac{A_s}{y_s}} = -D_1\gamma_s \,, \qquad \beta_{\frac{A_s^*}{y_s^*}} = -\bar{D}_1\gamma_s \,, \qquad \beta_{\tilde{m}_s^2} = (\bar{D}_1 D_1 + D_2)\gamma_s$$

where

$$\begin{aligned} \gamma_s &= \sum_i \mathcal{P}_s^i \gamma_{\phi_i} \,, \\ \tilde{m}_s^2 &\equiv \mathcal{P}_s^i \tilde{m}_i^2 \,. \end{aligned} \tag{A.8}$$

and where

$$D_1 = -\sum_{m=1}^{N_I} \mathbf{H}_m \Big|_{\theta^2} \frac{\partial}{\partial \mathbf{H}_m} = -\sum_s \frac{A_s}{y_s} \frac{\partial}{\partial \ln y_s} \,, \qquad \bar{D}_1 = (D_1)^* \,,$$

and

$$D_2 = \sum_{m=1}^{N_I} \mathbf{H}_m \Big|_{\theta^4} \frac{\partial}{\partial \mathbf{H}_m} = \frac{1}{2}\tilde{m}_s^2 \sum_s \left(\frac{\partial}{\partial \ln y_s} + \frac{\partial}{\partial \ln y_s^*} \right) .$$

The proofs of these statements are essentially identical to those given above.

A.4 Mixing

If we have n fields which are not distinguished by any symmetry, we must extend the requirement of reparametrization invariance to include $\mathrm{U}(n)$ transformations

$$\phi \to e^{\mathbf{T}}\phi \,, \qquad \phi^\dagger \to \phi^\dagger e^{\mathbf{T}^\dagger} \,, \qquad \mathbf{Z} \to e^{-\mathbf{T}^\dagger}\mathbf{Z}e^{-\mathbf{T}} \,, \tag{A.9}$$

where now ϕ is an n component vector, and $\mathbf{T}$ and $\mathbf{Z}$ are n by n matrix valued superfields. Superpotential couplings involving ϕ fields transform like the appropriate tensors.

To simplify the construction of invariants under the transformation (A.9) it is convenient to define superfields $\mathbf{X}$ such that

$$\mathbf{Z} = \mathbf{X}^\dagger \mathbf{X} \,. \tag{A.10}$$

Under eq. (A.9),

$$\mathbf{X} \to \mathbf{X}e^{-\mathbf{T}} \,.$$

Note that there is an ambiguity in the definition of $\mathbf{X}$ since we could always redefine

$$\mathbf{X} \to U\mathbf{X} \tag{A.11}$$

where

$$U^\dagger U = 1 \,.$$

Thus physically meaningful equations must be covariant under the transformation (A.11), as well as (A.9).

We can now define a superfield

$$\Gamma \equiv \left(\mathbf{X} \frac{d\mathbf{X}^{-1}}{d\ln\mu} + \frac{d\mathbf{X}^{\dagger -1}}{d\ln\mu} \mathbf{X}^{\dagger} \right) \tag{A.12}$$

which under eq. (A.11) transforms as

$$\Gamma \to U \Gamma U^{\dagger} \,,$$

and is invariant under the transformation eq. (A.9). Provided that we choose at all values of μ

$$\mathbf{X}|_{\theta=0} = \mathbf{X}^{\dagger}|_{\theta=0} \tag{A.13}$$

then

$$\gamma = \Gamma|_{\theta=0} \tag{A.14}$$

is the usual anomalous dimension matrix. We refer to Γ as the anomalous dimension superfield.

If we write

$$\mathbf{Z} = e^{\mathbf{J}^{\dagger}} e^{-\theta^4 \tilde{m}^2} e^{\mathbf{J}} \,,$$

where $\mathbf{J}$ is an ordinary chiral superfield, then we can eliminate $\mathbf{J}$ by making a transformation (A.9) with

$$\mathbf{T} = \mathbf{J} \,.$$

Thus $\tilde{m}^2$ is the soft supersymmetry breaking mass squared matrix in a canonical basis. By choosing

$$\mathbf{X} = e^{-\theta^4 \tilde{m}^2/2} e^{\mathbf{J}} \,, \tag{A.15}$$

with

$$\mathbf{J}|_{\theta=0} = \mathbf{J}^{\dagger}|_{\theta=0} \,, \tag{A.16}$$

and examining eq. (A.12), we see that the soft mass term beta functions can be read off from

$$\frac{d\tilde{m}^2}{d\ln\mu} = \Gamma|_{\theta^4} \,. \tag{A.17}$$

To find the θ^4 component of Γ, we define couplings

$$\mathbf{y}_{sb_1\ldots b_n} = \hat{y}_{sa_1\ldots a_n} (\mathbf{X}^{-1})^{a_1}_{b_1} \ldots (\mathbf{X}^{-1})^{a_n}_{b_n} \,. \tag{A.18}$$

Here $\hat{y}_{sa_1\ldots a_n}$ is the superpotential coefficient of $\phi_s \phi_{a_1} \ldots \phi_{a_n}$. Unlike the $\hat{y}$, the $\mathbf{y}$ couplings are not chiral superfields. However they are invariant under the transformation (A.9). Furthermore they transform covariantly under (A.11), and Γ is a covariant function of these couplings under the transformation eq. (A.11). Provided that we make the choices eqs. (A.15) and (A.16), then

$$\tilde{y} = \mathbf{y}|_{\theta=0}$$

is a superpotential coupling in a basis with canonically normalized kinetic terms and

$$\tilde{A} = \mathbf{y}|_{\theta^2}$$

is the associated trilinear scalar coupling. To find the $\tilde{m}^2$ beta function, we need to read off the θ^4 component of Γ in a canonical basis.

To do this, we can define

$$D_1 \equiv -\sum_s \tilde{A}_s \left(\frac{\partial}{\partial \tilde{y}_s}\right)$$

as before and D_2 as

$$D_2 \equiv \sum_s \frac{1}{2}\left((\tilde{m}^2)^{a_1}_{b_1}\delta^{a_2}_{b_2}...\delta^{a_n}_{b_n} + \text{perms of } a_i, b_i\right)\left(\tilde{y}_{sa_1...a_n}\frac{\partial}{\partial \tilde{y}_{sb_1...b_n}} + \tilde{y}_s{}^{*b_1...b_n}\frac{\partial}{\partial \tilde{y}_s{}^{*a_1...a_n}}\right). \tag{A.19}$$

Then we have the matrix equation

$$(\bar{D}_1 D_1 + D_2)\gamma = \frac{d\tilde{m}^2}{d\ln\mu}.$$

A.5 Four dimensional gauge couplings

Now let us turn our attention to a four-dimensional theory with a single gauge coupling, in order to focus on some of the special issues associated with this coupling. Let us consider a simple group G coupled to a single matter field ϕ in some representation r of G, with index equal to twice T_r. The gauge field strength is contained in a superfield

$$W_\alpha = -\frac{1}{4}\bar{D}\bar{D}e^{-V}D_\alpha e^V$$

and the classical lagrangian is given by

$$\int d^4\theta\ \phi^\dagger e^V \phi + \int d^2\theta\ \mathbf{S}_0\ \text{ tr }\ W_\alpha W^\alpha + \text{hermitean conjugate}$$

where the chiral background superfield

$$\mathbf{S}_0 = \frac{1}{2g_0^2}(1 - 2M_0\theta^2) - \frac{i\Theta}{16\pi^2}$$

contains the bare gauge coupling g_0, the bare gaugino mass M_0, and the theta angle. Unlike the other holomorphic couplings, $\mathbf{S}_0$ is renormalized at one loop

$$\mathbf{S}(\mu) = \mathbf{S}_0 + \frac{b_0}{16\pi^2}\log\frac{\mu}{\mu_0}$$

where μ_0 is the scale at which g_0 is defined, and $b_0 = 3T_G - T_r$ is the coefficient of the one-loop beta function. Letting

$$\hat{\Lambda}^{b_0} \equiv \mu_0^{b_0} e^{-\frac{8\pi^2}{g_0^2}}$$

generalize to a superfield

$$\hat{\mathbf{\Lambda}}^{b_0} \equiv \mu_0^{b_0} e^{-16\pi^2 \mathbf{S}_0} = \mu^{b_0} e^{-16\pi^2 \mathbf{S}}$$

we have

$$\mathbf{S}(\mu) = \frac{b_0}{16\pi^2}\log\frac{\mu}{\hat{\mathbf{\Lambda}}}$$

Note

$$\mathbf{S}(\mu)|_{\theta^2} = \mathbf{S}_0|_{\theta^2}$$

so the running holomorphic gaugino mass divided by the running holomorphic gauge coupling is a renormalization group invariant.

Under the $\mathrm{U}(1)_\phi$ symmetry (A.2), $\mathbf{S}$ transforms anomalously,

$$\mathbf{S}_0 \to \mathbf{S}_0\,, \qquad \mathbf{S} \to \mathbf{S} - \frac{T_r}{8\pi^2}\mathbf{T}\,, \qquad \hat{\mathbf{\Lambda}}^{b_0} \to \hat{\mathbf{\Lambda}}^{b_0} e^{2T_r \mathbf{T}} \tag{A.20}$$

An invariant under this symmetry is

$$\mu^{2b_0} e^{-16\pi^2 \mathbf{R}_0} = \left[\hat{\mathbf{\Lambda}}^{b_0}\right]^\dagger \mathbf{Z}^{-2T_r} \hat{\mathbf{\Lambda}}^{b_0}\,.$$

However, the logarithm of this invariant is not the physical gauge coupling, as it has the wrong two-loop beta function. A suitable coupling may defined using the NSVZ scheme [31, 32, 33]. NSVZ define a different invariant, $\mathbf{R}$, through the instanton factor

$$\mu^{2b_0} e^{-16\pi^2 \mathbf{R}} = \left[\hat{\mathbf{\Lambda}}^{b_0}\right]^\dagger \mathbf{Z}^{-2T_r} \mathbf{Z}_\lambda^{2T_G} \hat{\mathbf{\Lambda}}^{b_0}\,.$$

where $\mathbf{Z}_\lambda$ is the gaugino wave-function renormalization. (The reason $\mathbf{Z}_\lambda$ appears with a positive sign is that the gaugino zero modes come with a negative sign but the more numerous gauge-boson zero modes more than compensate for this.) Taking the logarithm we obtain

$$\mathbf{R} = \mathbf{S} + \mathbf{S}^\dagger - \frac{1}{8\pi^2}\left(T_r \ln \mathbf{Z} - T_G \ln \mathbf{Z}_\lambda\right) \tag{A.21}$$

Using

$$\mathbf{Z}_\lambda = \mathbf{R}$$

which follows from the fact that the gauge coupling and the wave function renormalization of the gaugino must be equal by supersymmetry, we may also write this as

$$\mathbf{R} - \frac{T_G}{8\pi^2} \ln \mathbf{R} = \mathbf{S} + \mathbf{S}^\dagger - \frac{1}{8\pi^2} T_r \ln \mathbf{Z} = \mathbf{R}_0 \tag{A.22}$$

We will soon need the fact that

$$\frac{\partial \mathbf{R}_0}{\partial \ln \mu} = \left(1 - \frac{T_G}{8\pi^2 \mathbf{R}}\right) \frac{\partial \mathbf{R}}{\partial \ln \mu} = \frac{1}{8\pi^2}\left(-T_r \frac{\partial \ln \mathbf{Z}}{\partial \ln \mu}\right) \tag{A.23}$$

Let us now define some natural objects using these invariants. We may write

$$4\pi \mathbf{R} = \alpha^{-1}(1 - M\theta^2 - M^* \bar{\theta}^2 - P\theta^4)$$

where $\alpha = g^2/4\pi$ is the physical gauge coupling and M is the physical gaugino mass; P, also invariant, is not an independent quantity as we will see in a moment. The gauge coupling

$$\alpha^{-1} = 4\pi \mathbf{R}|_{\theta=0}$$

has beta function

$$\beta_{\alpha^{-1}} = 4\pi \frac{\partial \mathbf{R}}{\partial \ln \mu}\Big|_{\theta=0}$$

which is related to the anomalous dimension of the gluino

$$\gamma_\lambda \equiv -\frac{\partial \ln \mathbf{R}}{\partial \ln \mu}\Big|_{\theta=0}$$

by

$$\gamma_\lambda = -\alpha\beta_{\alpha^{-1}} = \beta_{\ln\alpha} = 2\frac{\beta_\lambda}{g} = \frac{\beta_\alpha}{\alpha}.$$

From eq. (A.22) and eq. (A.23) we obtain

$$\gamma_\lambda = -\frac{b_0 + T_r\gamma_\phi}{\frac{2\pi}{\alpha} - T_G}\,.$$

The gaugino mass satisfies

$$M = -\ln \mathbf{R}\Big|_{\theta^2} \tag{A.24}$$

from which we learn

$$\beta_M = -\frac{\partial \ln \mathbf{R}}{\partial \ln \mu}\Big|_{\theta^2} = \frac{\partial \gamma_\lambda}{\partial \mathbf{R}}\,\mathbf{R}|_{\theta^2} = -\frac{M}{\alpha}\frac{\partial \gamma_\lambda}{\partial \alpha^{-1}} \equiv D_1\gamma_\lambda$$

where in the last steps we have used the fact that $\mathbf{R}$ is an invariant and can only depend on invariant quantities.[3] Note that if this is the only coupling, a solution to this equation is $M \propto \gamma_\lambda$.

Finally, eq. (A.21) shows

$$\frac{P}{\alpha} = -4\pi\mathbf{R}|_{\theta^4} = \frac{1}{2\pi}(T_r \ln \mathbf{Z} - T_G \ln \mathbf{R})|_{\theta^4} = \frac{1}{2\pi}\left(-T_r\tilde{m}^2 - T_G\left[-P - |M|^2\right]\right)$$

or

$$P = \frac{T_G|M|^2 - T_r\tilde{m}^2}{\frac{2\pi}{\alpha} - T_G} \equiv -\alpha\tilde{m}_\lambda^2\,. \tag{A.26}$$

Now, again using the fact that $\mathbf{R}$ is an invariant and can only depend on invariants,

$$\beta_{\tilde{m}^2} = -\frac{\partial \ln \mathbf{Z}}{\partial \ln \mu}\Big|_{\theta^4} = \left|\mathbf{R}|_{\theta^2}\frac{\partial \gamma_\phi}{\partial \mathbf{R}}\right|^2 + \mathbf{R}|_{\theta^4}\frac{\partial \gamma_\phi}{\partial \mathbf{R}} \equiv (\bar{D}_1 D_1 + D_2)\gamma_\phi$$

where D_1 is as above, $\bar{D}_1 = D_1^*$, and

$$D_1 = -\frac{M}{\alpha}\frac{\partial}{\partial\alpha^{-1}}\,,\ \ \bar{D}_1 = D_1^*\,,\ \ D_2 = \tilde{m}_\lambda^2\frac{\partial}{\partial\alpha^{-1}}$$

[3] Alternatively we may observe that

$$\frac{M}{\alpha} = -4\pi\mathbf{R}|_{\theta^2} = \frac{1}{2\pi}\left(M_0 b_0 \log\frac{\mu_0}{\Lambda} + T_r \ln \mathbf{Z}|_{\theta^2} + T_G M\right) \tag{A.25}$$

which gives

$$\frac{\partial}{\partial \ln \mu}\left[M\left(\frac{2\pi}{\alpha} - T_G\right)\right] = \beta_M\left[\frac{2\pi}{\alpha} - T_G\right] + 2\pi M\beta_{\alpha^{-1}} = T_r\frac{\partial \ln \mathbf{Z}}{\partial \ln \mu}\Big|_{\theta^2} = -T_r D_1\gamma_\phi\,.$$

Since

$$-T_r D_1\gamma_\phi = D_1\left(\gamma_\lambda\left[\frac{2\pi}{\alpha} - T_G\right]\right) = \left[\frac{2\pi}{\alpha} - T_G\right]D_1\gamma_\lambda + \frac{2\pi M}{\alpha}\gamma_\lambda$$

we see again that $\beta_M = D_1\gamma_\lambda$.

Thus

$$\beta_{\tilde m^2} = \left[\left| \frac{M}{\alpha} \right|^2 \frac{\partial^2}{\partial(\alpha^{-1})^2} - \frac{T_G|M|^2 - T_r\tilde m^2}{8\pi^2\left(1-\frac{T_G\alpha}{2\pi}\right)} \frac{\partial}{\partial(\alpha^{-1})} \right] \gamma_\phi$$

in agreement with the literature.

A.6 General four-dimensional theory

It is clear now how to write the beta functions for physical parameters in a general theory. A general theory has invariants $\mathbf{I}_m$, of the form

$$\ln \mathbf{I}_m = \ln\sigma_m + \ln\sigma_m^* + \theta^2\omega_m + \bar\theta^2\omega_m^* + \theta^4\nu_m$$

If

$$\mathcal{B}_{\ln \mathbf{I}_m} = \Gamma_m$$

then

$$\beta_{\ln\sigma_m} = \frac{1}{2}\gamma_m\ , \ \beta_{\omega_m} = -D_1\gamma_m\ , \ \beta_{\nu_m} = (D_1^* D_1 + D_2)\gamma_m \tag{A.27}$$

where

$$-D_1 = \omega_n \frac{\partial}{\partial \ln\sigma_n}\ , \qquad D_2 = \nu_n\frac{\partial}{\partial\ln\sigma_n} = \frac{1}{2}\nu_n\left(\frac{\partial}{\partial\ln\sigma_n} + \frac{\partial}{\partial\ln\sigma_n^*}\right) \tag{A.28}$$

As a check, let us verify that we obtain the results previously in the literature for the case of a single gauge coupling and a single Yukawa coupling. We find

$$\begin{aligned} D_1^*D_1 &= \left|\frac{M}{\alpha}\right|^2 \frac{\partial^2}{\partial(1/\alpha)^2} + \frac{M^*}{\alpha}\frac{A}{y}\frac{\partial^2}{\partial(1/\alpha)\ \partial\ln y} + \frac{M}{\alpha}\frac{A^*}{y^*}\frac{\partial^2}{\partial(1/\alpha)\ \partial\ln y^*} + \left|\frac{A}{y}\right|^2 \frac{\partial^2}{\partial\ln y\ \partial\ln y^*} \\ &= |M\alpha|^2\left(\frac{\partial^2}{\partial\alpha^2} + \frac{2}{\alpha}\frac{\partial}{\partial\alpha}\right) - M^*\alpha\frac{A}{y}\frac{\partial^2}{\partial\alpha\ \partial\ln y}M\alpha\frac{A^*}{-y^*}\frac{\partial^2}{\partial\alpha\ \partial\ln y^*} + \left|\frac{A}{y}\right|^2\frac{\partial^2}{\partial\ln y\ \partial\ln y^*} \end{aligned}$$

while

$$\begin{aligned} D_2 &= -\alpha^2\tilde m_g^2\frac{\partial}{\partial\alpha} + \frac{1}{2}\tilde m_y^2\left(\frac{\partial}{\partial\ln y} + \frac{\partial}{\partial\ln y^*}\right) \\ &= \frac{\alpha^2}{2\pi}\frac{T_G|M|^2 - T_r\tilde m^2}{1-\frac{T_G\alpha}{2\pi}}\frac{\partial}{\partial\alpha} + \frac{1}{2}\tilde m_y^2\left(\frac{\partial}{\partial\ln y} + \frac{\partial}{\partial\ln y^*}\right)\ . \end{aligned}$$

B. Stability of supersymmetric fixed points

We now prove that at a non-trivial conformal fixed point, all A terms and gaugino masses associated with fixed non-zero couplings and all combinations of scalar masses $\tilde m_s^2 \equiv \sum_i \mathcal{P}_s^i \tilde m_i^2$ run to zero.

A conformal fixed point is characterized by some non-zero couplings $\sigma_r = \sigma_{r*}$ satisfying the conditions

$$\beta_{\ln\sigma_r}(\sigma_v = \sigma_{v*}) = d^0_{\sigma_r}$$

where d^0_σ is the engineering dimension of σ [see eq. (A.1) and surrounding discussion.] More generally, near such a fixed point each beta-function superfield satisfies

$$\mathcal{B}_r = d^0_{\mathbf{I}_r} + \left(\frac{\partial \mathcal{B}_r}{\partial \ln \mathbf{I}_s}\right)_* (\ln \mathbf{I}_s - \ln \mathbf{I}_{s*}) + \cdots$$

where a subscript $*$ implies that the quantity is to be evaluated *at* the fixed point. Since the fixed point is supersymmetric,

$$\mathbb{\Gamma}'_{rs} \equiv \left(\frac{\partial \mathcal{B}_r}{\partial \ln \mathbf{I}_s}\right)\Bigg|_{\ln \mathbf{I}_v = \ln \sigma_{v*}} = \left(\frac{\partial \gamma_r}{\partial \ln \sigma_s}\right)_* \tag{B.1}$$

is a simple matrix of numbers. (Note σ_{v*} is a constant times the simple power of the renormalization scale μ needed to make it dimensionless; this $\mu^{-d^0_s}$ dependence does not appear in the derivative with respect to $\ln \sigma_s$.)

We are only interested in stable fixed points — that is, fixed points with the property that for all nonzero σ_r, any nearby theory with $\ln \sigma_r = \ln \sigma_{r*} + \delta_r$ flows back to the original fixed point. This implies

$$\sum_r \delta_r \ \beta_{\ln \sigma_r}\Bigg|_{\ln \sigma_v = \ln \sigma_{v*} + \delta_v} > 0$$

which requires, since the δ_r are arbitrary, that

$$\frac{\partial \beta_{\ln \sigma_r}}{\partial \ln \sigma_s}\Bigg|_{\sigma_v = \sigma_{v*}}$$

be a matrix whose eigenvalues are all positive definite, or equivalently that $\mathbb{\Gamma}'$ defined in eq. (B.1) be a positive matrix.

However, the same matrix $\mathbb{\Gamma}'$ controls the running of the A-terms and gaugino masses through eq. (A.27)–(A.28). In particular, $\beta_{\omega_s} = -D_1 \gamma_s$, which contains $\mathbb{\Gamma}'$. The positivity of $\mathbb{\Gamma}'$ required for stability thus drives the A-terms and gaugino masses to zero. Similarly, once the ω_s have run small, the appearance of $\mathbb{\Gamma}'$ in $D_2 \gamma_s$ ensures the linear combinations of soft masses appearing in the ν_s also run small.

This can be stated in superfield language. The stability of the fixed point requires that $\mathbf{I}_s \to \mathbf{I}_{s*}$ in the infrared. Since $\mathbf{I}_{s*}$ has no θ^2 or θ^4 terms, any supersymmetry violation in $\mathbf{I}_s$, through ω_s and ν_s, must approach zero in the infrared.[4]

Notice however that the running of the ν_s to zero does not necessarily guarantee that all soft scalar masses run toward zero. Recall that ν_s refers to the soft masses associated to *coupling constants*; in particular for each Yukawa coupling y_s the combination $\tilde{m}^2_s$ defined in eq. (A.8) runs to zero, while for gauge couplings the combination $\tilde{m}^2_\lambda$ defined in eq. (A.26) runs to zero. Given a scalar field ϕ_i, its soft mass $\tilde{m}^2_i$ runs to zero only if

[4]Nota Bene: these equations do not imply that the ω_s actually *reach* zero. Although ω_s has positive anomalous dimension, it need not have dimension greater than one. This means that at some scale μ and ω_s are of the same order; below this scale the above equations are no longer valid. In our models this issue is inconsequential, since these equations are only used at scales far higher than 1 TeV.

$\tilde{m}_i^2$ can be written as a linear combination of the $\tilde{m}_s^2$ and $\tilde{m}_\lambda^2$. This will be an essential constraint on model building. The following simple counting argument shows that this constraint is satisfied provided the strong couplings of the theory break all global non-R U(1) symmetries.

In general, with n irreducible multiplets of chiral superfields (irreducible under non-abelian gauge or global symmetries) of chiral superfields, we have n independent soft scalar mass squared terms, and n possible independent non-R U(1) global symmetries. We will now show that a theory which explicitly breaks all these symmetries via gauge anomalies or superpotential terms will drive all soft SUSY-breaking scalar mass terms to zero.

The linear combination of soft scalar masses in eq. (A.26), which run to zero if the associated gauge coupling is at a stable fixed point, is precisely the same as the linear combination of the corresponding U(1)'s which is anomalous under the corresponding gauge transformation. Furthermore, each superpotential coupling is also associated through eq. (A.8) with a linear combination of scalar masses-squared, which is again exactly the linear combination which is explicitly broken by the corresponding superpotential coupling. If all n possible non-R U(1) symmetries are explicitly broken, by gauge anomalies and superpotential couplings, there must be at least n couplings which break independent combinations of the possible symmetries. Then there must be n independent linear combinations of the scalar masses-squared which run to zero. This is only possible if all scalar masses-squared run to zero.

An even simpler argument shows that any exact non-R U(1) symmetry of the strong couplings corresponds to a linear combination of scalar masses which do not run at the fixed point. Any such global symmetries could be gauged with arbitrarily weak coupling, provided spectator chiral superfields are added to cancel anomalies. Now any soft SUSY-breaking mass squared terms proportional to the U(1) charge can be written as a supersymmetric Fayet-Iliopoulos term, plus soft terms for the spectators and a SUSY-breaking constant added to the hamiltonian. We have already argued that at the fixed point the SUSY-breaking gaugino masses, A terms, and scalar masses not proportional to global symmetry charges run to zero. Thus, up to spectator fields, and constant terms in the hamiltonian, at the fixed point the theory with such soft breaking terms is equivalent to a supersymmetric theory with Fayet-Iliopoulos terms and arbitrarily weak U(1) gauge couplings. Fayet-Iliopoulos terms are known not to get renormalized [34, 35].

C. Flow of supersymmetry breaking terms for the MSSM coupled to a conformal field theory

We now consider the effect on the soft parameters of the MSSM when we couple its fields to those of a conformal sector. Specifically, we study the energy regime near and below $M_>$ and above $M_<$. We will generally assume that at the scale $M_<$ the trilinear scalar terms are of order A_0, the gaugino masses are of order $m_{1/2}$, and the scalar masses-squared are of order m_0^2; for simplicity of presentation we will take these parameters to be of similar size, of order $M_{\rm SUSY}$. We will show that[5]

- The gaugino masses associated with the conformal-sector gauge group and and the A-terms with the strong superpotential couplings involving the conformal sector and its couplings to the standard model fields are driven down to a size $\alpha M_{\rm SUSY}/4\pi$, where α is a standard model gauge coupling. These are then too small to have a large impact on the standard model sector.

- The third generation of the standard model, although it retains large Yukawa couplings to the Higgs boson and corresponding large A-terms, does not contribute dominantly to the running of the other soft-breaking parameters, since it decouples from the strong sector except through two-loop effects.

- The standard model A-terms largely run by the same factor as their corresponding Yukawa couplings, and the gaugino masses tend to run by a similar factor as the the gauge couplings α.

- The soft masses of the third generation do not run by large factors. However, the soft masses of the first two generations, which are governed by the conformal sector, are driven to values of order $\tilde{m}^2 \sim \alpha |M|^2/4\pi$, where M is the gluino mass. These values would be zero were it not for the standard model couplings.

Thus, we claim that the after the conformal sector decouples, the third generation soft parameters (more precisely, the top-quark and Higgs parameters and some, or possibly all, of the bottom and tau parameters) are at a scale not too far from $M_{\rm SUSY}$; the gaugino masses lie only a little below this scale; the matrices of A-terms have roughly inherited the hierarchical structure of the Yukawa couplings with which they are associated, suppressing their effects on flavor-changing neutral currents; and the soft masses of the first two generations of sfermions are of order $\tilde{m}^2 \sim \alpha |M|^2/4\pi$, small enough that the non-degeneracies among them will not lead to large flavor-changing neutral currents after the additive renormalization of eq. (5.1).

Our approximate conformal fixed point has three different types of operators which we must consider. First, there are irrelevant operators which control the flow into the fixed point; these are characterized by positive eigenvalues in the matrix $\mathbb{\Gamma}'$, defined in eq. (B.1). Second, there are couplings which run slowly, such as the standard model gauge couplings, or the top quark Yukawa coupling. These are characterized by eigenvalues in the matrix $\mathbb{\Gamma}'$ which are very small. We will refer to these as weak marginal couplings, though they are only approximately marginal. Finally, there is at least one relevant operator whose eigenvalue in $\mathbb{\Gamma}'$ is large and negative, but whose associated coupling is very small initially. It is this operator which drives the theory away from the CFT when its coupling grows sufficiently large.

[5] In this list we crudely separate the first two generations from the third; however, as discussed in [1] we may well expect part of the third generation to be strongly coupled to the conformal sector and to therefore be more characteristic of the first two generations.

In the previous section we showed that if the relevant and nearly marginal operators are set to zero, so that Γ' for the nonzero couplings is a positive matrix, the approach to the fixed point drives all irrelevant couplings and their supersymmetric partners to zero. The flow is characterized by the eigenvalues of Γ', with the smallest eigenvalue, or equivalently the least irrelevant operator, determining the flow at low energy.

In particular, we showed that for the nonzero couplings σ_{m*} in the strong sector, we have

$$(\dot{\ln \sigma})_r - (\Gamma')_{rs} \ln \sigma_s = \text{order}[(\sigma - \sigma_*)^2] \tag{C.1}$$

for the couplings σ_r themselves,

$$\dot{\omega}_r - (\Gamma')_{rs}\omega_s = 0 \tag{C.2}$$

for ω_r, the associated strong-coupling A-terms and gaugino masses, and

$$\dot{\nu}_r - (\Gamma')_{rs}\nu_s = (\omega_u^* \cdot \frac{\partial}{\partial \ln \sigma_u^*})(\omega_v \cdot \frac{\partial}{\partial \ln \sigma_v})\gamma_r \tag{C.3}$$

for ν_r, the associated combinations of soft scalar masses-squared. In the final approach to the CFT, then, only the least irrelevant coupling and its associated supersymmetry-breaking couplings will be nonzero. The holomorphic soft terms are driven to zero with the same power law as the coupling is driven to its fixed value. The soft masses-squared have a "driving term" from the holomorphic terms, but this shrinks quickly to zero, leading them to be driven to zero also with the same power law.

However, now consider the effect of the weak and nearly marginal standard-model couplings $\sigma_\bullet$. (We will use subscript m, n to indicate couplings in the strong sector and $\bullet$ to indicate weak marginal ones from the standard model sector.) Since they are rather small and slowly running, we may treat them perturbatively. In particular, standard model Yukawa and gauge couplings $y_\bullet, g_\bullet$ will appear in the anomalous dimensions γ_m. On general grounds,

$$\frac{\partial \gamma_r}{\partial \ln y_\bullet} \lesssim \text{order}\left(\frac{|y_\bullet|^2}{16\pi^2}\right); \qquad \frac{\partial \gamma_r}{\partial(1/\alpha_\bullet)} \lesssim \text{order}\left(\frac{\alpha_\bullet^2}{4\pi}\right). \tag{C.4}$$

Thus the contribution of the weak couplings to the flow of the strong couplings is indeed small. One might be concerned that third-generation Yukawa couplings to the Higgs boson might be too large for this conclusion. However, any fields which have large Yukawa couplings at low energy must, in the conformal regime, have small anomalous dimensions. Such fields typically couple to the conformal sector through standard model gauge couplings and irrelevant operators which run to small values inside the conformal regime. For the top quark, for example, it must be that

$$\frac{\partial \gamma_r}{\partial \ln y_t} \lesssim \text{order}\left(\frac{\alpha_\bullet^2 |y_t|^2}{32\pi^3}\right) \tag{C.5}$$

since by the end of the conformal regime the top quark must couple to the conformal sector only at two standard model loops. In short, all entries in $\mathbb{\Gamma}'$ involving derivatives with respect to standard model couplings are small.[6]

These derivatives come into the strong-coupling flow equations (C.1)–(C.3) as nearly-constant and small "driving terms" on the right-hand sides.

$$\dot{\ln\sigma_r} - (\mathbb{\Gamma}')_{rs}\ln\left(\frac{\sigma_s}{\sigma_{s*}}\right) = (\mathbb{\Gamma}')_{r,\bullet}\ln\sigma_\bullet + \text{order}\left[(\sigma_s - \sigma_{s*})^2\right] \tag{C.6}$$

for the couplings themselves,

$$\dot{\omega_r} - (\mathbb{\Gamma}')_{rs}\omega_s = (\mathbb{\Gamma}')_{r,\bullet}\omega_\bullet \tag{C.7}$$

for the associated strong-coupling A-terms and gaugino masses, and

$$\dot{\nu_r} - (\mathbb{\Gamma}')_{rs}\nu_s = (\mathbb{\Gamma}')_{r,\bullet}\nu_\bullet + \left(\omega^*_\bullet \cdot \frac{\partial}{\partial\ln\sigma^*_\bullet}\right)\left(\omega'_\bullet \cdot \frac{\partial}{\partial\ln\sigma_{\bullet'}}\right)\gamma_r + \text{order}\left(\omega_s^2, \omega_s\omega_\bullet\right) \tag{C.8}$$

for the soft scalar masses. In short, the weak couplings and associated soft terms serve as weak driving forces — of order $\alpha_\bullet^2/4\pi$ or of order $|y|^2/16\pi^2$ for light fermions — on the damped strong couplings and associated soft terms. To see how large these driving forces are, we must also understand what happens to the soft terms $\omega_\bullet$ and $\nu_\bullet$.

The couplings which do not go to fixed values have beta functions which are not controlled by the matrix $\mathbb{\Gamma}'$. They are simply

$$\beta_{\ln\sigma_\bullet} = \frac{1}{2}\gamma_\bullet\,.$$

For all standard model couplings, these anomalous dimensions must be positive in the limit all standard model gauge couplings are zero, by unitarity; and so, if negative, must be of order $-\alpha_\bullet$. However, if positive, they may be of order one, in which case the associated couplings run quickly toward zero. This fact was used to obtain the fermion mass hierarchy in [1]. In such models, the light fermions have large anomalous dimensions near the fixed point, and their associated Yukawa couplings $y_\bullet$ have large positive beta functions.

By contrast, the soft terms are still controlled by $\mathbb{\Gamma}'$. For A-terms and gaugino masses associated to such couplings (which include the usual MSSM A-terms and gaugino masses)

$$\dot{\omega}_\bullet = (\mathbb{\Gamma}')_{\bullet,\bullet'}\omega_{\bullet'} + (\mathbb{\Gamma}')_{\bullet,r}\omega_r \tag{C.9}$$

The first term on the right-hand side is small because $(\mathbb{\Gamma}')_{\bullet,\bullet'}$ is perturbative in standard model couplings. The second term will be small because ω_r is only nonzero (in the conformal regime) due to the driving force in (C.7), which is proportional to weak couplings. Thus, $\omega_\bullet$ runs slowly, which implies that for standard model gauge and Yukawa couplings, M/α and A/y are nearly renormalization-group invariant in the conformal regime. Thus,

[6] This is not absolutely guaranteed. If, say, the right-handed top quark superfield is coupled by a coupling h to an operator $\mathcal{O}$ in the conformal sector with dimension very close to two, then h will run slowly and (C.5) need not be true. We will assume for simplicity that $\dim\mathcal{O}$ is large enough that h becomes small somewhere inside the conformal regime.

for those couplings which run slowly, the A-terms and gaugino couplings will remain of the same order as their initial values. However, for those Yukawa couplings $y_\bullet$ which in our models run rapidly to zero due to large anomalous dimensions, the couplings $A_\bullet$ run quickly also, at nearly the same rate. This gives us our first important result: *all A-terms for light-fermion Yukawa couplings are roughly proportional to the associated Yukawa couplings.* The reason for the proportionality is "rough" is that our initial conditions in the ultraviolet have no special structure. We assume only that all Yukawa couplings (A-terms) are of the same order in the far ultraviolet, with no particular relations among them. The lack of precise proportionality leads to the flavor-violating signatures discussed in section 6.

We may now return to eq. (C.7). The driving term on the right-hand side contains $\omega_\bullet$, which as we have just seen is always of order $M_{\rm SUSY}$ times weak couplings. From (C.4) and (C.5) it is easily seen that the driving term in (C.7) is at most of order $(\alpha_\bullet/4\pi)M_\bullet$, where $M_\bullet$ is a standard model gaugino mass. Since in the strong sector $(\Gamma')_{rs} \sim 1$, each ω_r is driven to an energy-independent value

$$\omega_r \sim (\Gamma')_{r\bullet}\omega_\bullet \sim \frac{\alpha_\bullet}{4\pi}M_\bullet\,. \tag{C.10}$$

This shows our earlier assumption that ω_r makes a small contribution to the running of standard model soft terms $\omega_\bullet$ is consistent.

Similarly, it is now straightforward to see that the soft scalar masses-squared associated to the strong couplings will therefore not run to zero, but will instead reach approximate fixed points due to the small driving force from the second term in eq. (C.8)

$$\nu_r \sim \frac{\alpha_\bullet}{4\pi}M_\bullet^2 \tag{C.11}$$

where again $M_\bullet$ is a standard model gaugino mass.[7] Thus, the effect of the nonzero standard model couplings is to prevent the strong-coupling fixed point from driving the strong-coupling A-terms, gaugino masses, and soft scalar masses strictly to zero. For our purposes, the most important result is the last one: *the soft scalar masses for light fermions are not driven strictly to zero, but instead are each driven to values of order $\alpha_\bullet/4\pi$ times the square of the gaugino mass. Furthermore, these remaining small terms have no flavor symmetry relating them to one another.* This means that flavor symmetry is not exactly restored at the fixed point, even in models where for $\alpha_\bullet = y_\bullet = 0$ the CFP would have driven all standard model scalar masses to zero. This will also lead to potentially observable flavor-violating effects, as we discuss in section 6.

As argued in [1], at a CFP we can use the accidental infrared R invariance to define a basis for the matter superfields of the standard model. We now discuss the fixed point running of the terms in the scalar mass matrices which are off diagonal in the basis of fields with definite fixed point R charge. We will argue that the result is very simple:

[7]Note that it is always α_3 and M_3 which appear here, even when the sfermion whose mass is in question is color-neutral; this is because the anomalous dimensions of standard model fields X contain strongly-coupled loops with colored fields Q in them, so derivatives of the anomalous dimensions with respect to α_3 are of order α_3 with no additional suppression. However, there is not much difference between the standard model gauge couplings at the scale $M_<$ where this is to be evaluated.

these run rapidly to zero and, when the other couplings are at their supersymmetric fixed point values, the off-diagonal mass-squared terms run *only* via wave function renormalization.

The general result for the running of the non-holomorphic mass squared terms is

$$\beta_{\tilde{m}^2{}^i_j} = (\bar{D}_1 D_1 + D_2)\gamma^i_j\,. \tag{C.12}$$

At first one might be concerned that at the fixed point an offdiagonal mass term does not run at all, since its beta function is proportional to derivatives with respect to coupling constants of off-diagonal anomalous dimensions, and the latter vanish at the fixed point. However a subtlety of eq. (C.12) is that the operators D_1 and D_2 contain all *possible* couplings, whether or not these vanish. Usually it is sufficient to consider only the nonzero couplings, however in some cases turning on a coupling δ could change the anomalous dimension matrix by an amount which is linear in δ, and so partial derivatives of γ^i_j with respect to δ do not vanish. In particular the operator D_2 contains a contribution from superpotential couplings

$$D_2 = \text{gauge terms} + \frac{1}{2}(\tilde{m}^2)^i_j\left(y_{ia_1\ldots a_n}\frac{\partial}{\partial y_{ja_1\ldots a_n}} + \text{perms} + y^{*ja_1\ldots a_n}\frac{\partial}{\partial y^{*ia_1\ldots a_n}} + \text{perms}\right).$$

Here $y_{ia_1\ldots a_n}$ is the superpotential coefficient of $\phi_i\phi_{a_1}\ldots\phi_{a_n}$. In evaluating D_2, all superpotential couplings $y_{ia_1\ldots a_n}$ must be considered which could produce contributions to γ^i_j which are linear in $y_{ia_1\ldots a_n}$. Assuming all terms other than the $\tilde{m}^2$ have reached their fixed point values (which means that D_1 and the gauge contribution to D_2 vanish) we can find the beta function for $(\tilde{m}^2)^i_j$. For the case we are interested in ϕ_i, ϕ_j are standard model superfields with superpotential couplings

$$y_i\phi_i\mathcal{O}_i + y_j\phi_j\mathcal{O}_j$$

with y_i, y_j nonvanishing at the fixed point. The $\mathcal{O}_i$ are operators of the superconformal sector with *different* R charges. At the fixed point ϕ_i acquires a non vanishing anomalous dimension γ^i_i, and the anomalous dimension of ϕ_j is γ^j_j. If we turn on infinitesimal couplings

$$\delta_j\phi_j\mathcal{O}_i + \delta_i\phi_i\mathcal{O}_j$$

then we slightly perturb the fixed point and different linear combinations of fields will acquire definite anomalous dimensions. For infinitesimal δ's, $\phi_i + \frac{\delta_j}{y_i}\phi_j$ and $\phi_j + \frac{\delta_i}{y_j}\phi_i$ will diagonalize the anomalous dimension matrix and will get anomalous dimensions γ^i_i and γ^j_j respectively. Hence,

$$\frac{\partial\gamma^i_j}{\partial\delta_j} = \frac{1}{y_i}\gamma^i_i\,,$$

and

$$\frac{\partial\gamma^i_j}{\partial\delta_i} = \frac{1}{y_j}\gamma^j_j\,.$$

These do not vanish at the fixed point. Thus at the fixed point

$$\beta_{\tilde{m}^2{}^i_j} = \frac{1}{2}(\tilde{m}^2)^i_j \left(y_i \frac{\partial \gamma^i_j}{\partial \delta_j} + y_j \frac{\partial \gamma^i_j}{\partial \delta_i} + \text{h.c.} \right) = \left(\tilde{m}^2\right)^i_j \left(\gamma^i_i + \gamma^j_j \right),$$

and the off-diagonal scalar mass terms run via wave-function renormalization only.

To summarize, we expect the following spectrum to emerge from the conformal regime, where the only substantial violation of conformal symmetry is due to third-generation Yukawa couplings and standard model gauge couplings. The standard model gaugino masses $M_\bullet$ are somewhere below $M_{\rm SUSY}$, probably of order $\alpha_\bullet M_{\rm SUSY}$. The standard model A-terms are roughly proportional to standard model Yukawa couplings times an overall scale A_0; for the light fermions, both the Yukawa coupling and the A-term are driven small at about the same rate. As seen in section 6, the overall scale A_0 of the A terms must be relatively low, which fortunately is not inherently unnatural. The conformal-sector A-terms and gaugino masses are suppressed far below $M_{\rm SUSY}$ by small standard model couplings and loop factors. The strong-sector combinations of soft-scalar masses-squared, *which by assumption include all of the standard model squark and slepton masses except a few members of the third generation*, are driven to small values of order $(\alpha_\bullet/4\pi)M_\bullet^2$. Thus, flavor violation among the partners of the light fermions has been naturally suppressed.

D. Consequences of escaping

At some point a certain relevant coupling in the model must drive the theory away from its fixed point. Since it is relevant, it is dimensionful at the fixed point. For illustration, let us take it to be an effective mass m for one or more particles. The dimension*less* coupling which characterizes its effect is m/μ, which is negligible at high scales. Once we approach the scale m, the coupling becomes important. As we pass through the mass threshold, there could be important renormalization effects. What must happen, or not happen, to ensure that flavor violation is not reintroduced?

Clearly, if more than one relevant operator is necessary to get rid of the fixed point and the couplings of the strong sector to the standard model, then flavor violation may be reintroduced. Any region of energy which is not conformal and in which strong flavor-violating couplings are still present in the superpotential will ruin the scenario. We therefore must require that the departure from the fixed point and removal of all couplings of the strong sector to the standard model be both rapid and complete. This is a model-building issue which we will not address further here; the requirement is not difficult to satisfy, as shown by explicit example in [1], so we do not consider it a major issue. We will instead argue that if these conditions are satisfied, the threshold effects are small enough that the previously discussed properties of our scenario are preserved.

The argument is simple. The strongly-coupled sector of the theory is nearly supersymmetric, since the conformal regime has driven the supersymmetry-breaking in the strong sector down to a scale $M_{\rm strong}$ which is much smaller than the original $M_{\rm SUSY}$. The escape sector is supersymmetric, and thus cannot increase M_{strong} unless there is an opportunity for a large renormalization group effect. If the escape sector does its work over a small range of energies then there is simply is no opportunity for a significant enhancement of supersymmetry breaking. At worst, flavor violation can be affected only at order one.

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